

# The **libcoin** Package

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# Chapter 1

## Introduction

The **libcoin** package implements a generic framework for permutation tests. We assume that we are provided with  $n$  observations

$$(\mathbf{Y}_i, \mathbf{X}_i, w_i, \text{block}_i), \quad i = 1, \dots, N.$$

The variables  $\mathbf{Y}$  and  $\mathbf{X}$  from sample spaces  $\mathcal{Y}$  and  $\mathcal{X}$  may be measured at arbitrary scales and may be multivariate as well. In addition to those measurements, case weights  $w_i \in \mathbb{N}$  and a factor  $\text{block}_i \in \{1, \dots, B\}$  coding for  $B$  independent blocks may be available. We are interested in testing the null hypothesis of independence of  $\mathbf{Y}$  and  $\mathbf{X}$

$$H_0 : D(\mathbf{Y} \mid \mathbf{X}) = D(\mathbf{Y})$$

against arbitrary alternatives. [Strasser and Weber \(1999\)](#) suggest to derive scalar test statistics for testing  $H_0$  from multivariate linear statistics of a specific linear form. Let  $\mathcal{A} \subseteq \{1, \dots, N\}$  denote some subset of the observation numbers and consider the linear statistic

$$\mathbf{T}(\mathcal{A}) = \text{vec} \left( \sum_{i \in \mathcal{A}} w_i g(\mathbf{X}_i) h(\mathbf{Y}_i, \{\mathbf{Y}_i \mid i \in \mathcal{A}\})^\top \right) \in \mathbb{R}^{PQ}. \quad (1.1)$$

Here,  $g : \mathcal{X} \rightarrow \mathbb{R}^P$  is a transformation of  $\mathbf{X}$  known as the *regression function* and  $h : \mathcal{Y} \times \mathcal{Y}^n \rightarrow \mathbb{R}^Q$  is a transformation of  $\mathbf{Y}$  known as the *influence function*, where the latter may depend on  $\mathbf{Y}_i$  for  $i \in \mathcal{A}$  in a permutation symmetric way. We will give specific examples on how to choose  $g$  and  $h$  later on.

With  $\mathbf{x}_i = g(\mathbf{X}_i) \in \mathbb{R}^P$  and  $\mathbf{y}_i = h(\mathbf{Y}_i, \{\mathbf{Y}_i, i \in \mathcal{A}\}) \in \mathbb{R}^Q$  we write

$$\mathbf{T}(\mathcal{A}) = \text{vec} \left( \sum_{i \in \mathcal{A}} w_i \mathbf{x}_i \mathbf{y}_i^\top \right) \in \mathbb{R}^{PQ}. \quad (1.2)$$

The **libcoin** package doesn't handle neither  $g$  nor  $h$ , this is the job of **coin** and we therefore continue with  $\mathbf{x}_i$  and  $\mathbf{y}_i$ .

The distribution of  $\mathbf{T}$  depends on the joint distribution of  $\mathbf{Y}$  and  $\mathbf{X}$ , which is unknown under almost all practical circumstances. At least under the null hypothesis one can dispose of this dependency by fixing  $\mathbf{X}_i, i \in \mathcal{A}$  and conditioning on all possible permutations  $S(\mathcal{A})$  of the responses  $\mathbf{Y}_i, i \in \mathcal{A}$ . This principle leads to test procedures known as *permutation tests*. The conditional expectation  $\boldsymbol{\mu}(\mathcal{A}) \in \mathbb{R}^{PQ}$  and covariance  $\boldsymbol{\Sigma}(\mathcal{A}) \in \mathbb{R}^{PQ \times PQ}$  of  $\mathbf{T}$  under  $H_0$  given all permutations  $\sigma \in S(\mathcal{A})$  of the responses are derived by [Strasser and Weber \(1999\)](#):

$$\begin{aligned} \boldsymbol{\mu}(\mathcal{A}) &= \mathbb{E}(\mathbf{T}(\mathcal{A}) \mid S(\mathcal{A})) = \text{vec} \left( \left( \sum_{i \in \mathcal{A}} w_i \mathbf{x}_i \right) \mathbb{E}(h \mid S(\mathcal{A}))^\top \right), \\ \boldsymbol{\Sigma}(\mathcal{A}) &= \mathbb{V}(\mathbf{T}(\mathcal{A}) \mid S(\mathcal{A})) \\ &= \frac{\mathbf{w} \cdot}{\mathbf{w} \cdot(\mathcal{A}) - 1} \mathbb{V}(h \mid S(\mathcal{A})) \otimes \left( \sum_{i \in \mathcal{A}} w_i \mathbf{x}_i \otimes w_i \mathbf{x}_i^\top \right) \\ &= \frac{1}{\mathbf{w} \cdot(\mathcal{A}) - 1} \mathbb{V}(h \mid S(\mathcal{A})) \otimes \left( \sum_{i \in \mathcal{A}} w_i \mathbf{x}_i \right) \otimes \left( \sum_{i \in \mathcal{A}} w_i \mathbf{x}_i \right)^\top \end{aligned} \quad (1.3)$$

where  $\mathbf{w} \cdot (\mathcal{A}) = \sum_{i \in \mathcal{A}} w_i$  denotes the sum of the case weights, and  $\otimes$  is the Kronecker product. The conditional expectation of the influence function is

$$\mathbb{E}(h \mid S(\mathcal{A})) = \mathbf{w} \cdot (\mathcal{A})^{-1} \sum_{i \in \mathcal{A}} w_i \mathbf{y}_i \in \mathbb{R}^Q$$

with corresponding  $Q \times Q$  covariance matrix

$$\mathbb{V}(h \mid S(\mathcal{A})) = \mathbf{w} \cdot (\mathcal{A})^{-1} \sum_{i \in \mathcal{A}} w_i (\mathbf{y}_i - \mathbb{E}(h \mid S(\mathcal{A}))) (\mathbf{y}_i - \mathbb{E}(h \mid S(\mathcal{A})))^\top.$$

With  $A_b = \{i \mid \text{block}_i = b\}$  we get  $\mathbf{T} = \sum_{b=1}^B T(\mathcal{A}_b)$ ,  $\boldsymbol{\mu} = \sum_{b=1}^B \boldsymbol{\mu}(\mathcal{A}_b)$  and  $\boldsymbol{\Sigma} = \sum_{b=1}^B \boldsymbol{\Sigma}(\mathcal{A}_b)$ .

Having the conditional expectation and covariance at hand we are able to standardize a linear statistic  $\mathbf{T} \in \mathbb{R}^{PQ}$  of the form (1.2). Univariate test statistics  $c$  mapping an observed linear statistic  $\mathbf{t} \in \mathbb{R}^{PQ}$  into the real line can be of arbitrary form. An obvious choice is the maximum of the absolute values of the standardized linear statistic

$$c_{\max}(\mathbf{t}, \boldsymbol{\mu}, \boldsymbol{\Sigma}) = \max \left| \frac{\mathbf{t} - \boldsymbol{\mu}}{\text{diag}(\boldsymbol{\Sigma})^{1/2}} \right|$$

utilizing the conditional expectation  $\boldsymbol{\mu}$  and covariance matrix  $\boldsymbol{\Sigma}$ . The application of a quadratic form  $c_{\text{quad}}(\mathbf{t}, \boldsymbol{\mu}, \boldsymbol{\Sigma}) = (\mathbf{t} - \boldsymbol{\mu}) \boldsymbol{\Sigma}^+ (\mathbf{t} - \boldsymbol{\mu})^\top$  is one alternative, although computationally more expensive because the Moore-Penrose inverse  $\boldsymbol{\Sigma}^+$  of  $\boldsymbol{\Sigma}$  is involved.

The definition of one- and two-sided  $p$ -values used for the computations in the **libcoin** package is

$$\begin{aligned} P(c(\mathbf{T}, \boldsymbol{\mu}, \boldsymbol{\Sigma}) \leq c(\mathbf{t}, \boldsymbol{\mu}, \boldsymbol{\Sigma})) & \quad (\text{less}) \\ P(c(\mathbf{T}, \boldsymbol{\mu}, \boldsymbol{\Sigma}) \geq c(\mathbf{t}, \boldsymbol{\mu}, \boldsymbol{\Sigma})) & \quad (\text{greater}) \\ P(|c(\mathbf{T}, \boldsymbol{\mu}, \boldsymbol{\Sigma})| \leq |c(\mathbf{t}, \boldsymbol{\mu}, \boldsymbol{\Sigma})|) & \quad (\text{two-sided}). \end{aligned}$$

Note that for quadratic forms only two-sided  $p$ -values are available and that in the one-sided case maximum type test statistics are replaced by

$$\min \left( \frac{\mathbf{t} - \boldsymbol{\mu}}{\text{diag}(\boldsymbol{\Sigma})^{1/2}} \right) \quad (\text{less}) \quad \text{and} \quad \max \left( \frac{\mathbf{t} - \boldsymbol{\mu}}{\text{diag}(\boldsymbol{\Sigma})^{1/2}} \right) \quad (\text{greater}).$$

This single source file implements and documents the **libcoin** package following the literate programming paradigm. The keynote lecture on literate programming by Donald E. Knuth given at useR! 2016 in Stanford very much motivated this little experiment.

# Chapter 2

## R Code

### 2.1 R User Interface

```
"libcoin.R" 3a≡
```

```
  < R Header 154a >  
  < LinStatExpCov 4 >  
  < LinStatExpCov1d 6 >  
  < LinStatExpCov2d 8 >  
  < vcov LinStatExpCov 10 >  
  < doTest 12 >  
  < Contrasts 14 >  
  ◇
```

The **libcoin** package implements two R functions, `LinStatExpCov()` and `doTest()` for the computation of linear statistics, their expectation and covariance as well as for the computation of test statistics and  $p$ -values. There are two interfaces: One (labelled “1d”) when the data is available as matrices  $X$  and  $Y$ , both with the same number of rows  $N$ . The second interface (labelled “2d”) handles the case when the data is available in aggregated form; details will be explained later.

```
< LinStatExpCov Prototype 3b > ≡  
  (X, Y, ix = NULL, iy = NULL, weights = integer(0),  
   subset = integer(0), block = integer(0), checkNAs = TRUE,  
   varonly = FALSE, nresample = 0, standardise = FALSE,  
   tol = sqrt(.Machine$double.eps))◇
```

Fragment referenced in 4, 17.

Uses: `block` 26f, 27b, `subset` 26ade, `weights` 25b.

$\langle \text{LinStatExpCov } 4 \rangle \equiv$

```
LinStatExpCov <-  
function(LinStatExpCov Prototype 3b)  
{  
  if (missing(X) && !is.null(ix) && is.null(iy)) {  
    X <- ix  
    ix <- NULL  
  }  
  
  if (missing(X)) X <- integer(0)  
  
  ## <FIXME> for the time being only!!! </FIXME>  
  ##   if (length(subset) > 0) subset <- sort(subset)  
  
  if (is.null(ix) && is.null(iy))  
    .LinStatExpCov1d(X = X, Y = Y,  
                    weights = weights, subset = subset,  
                    block = block, checkNAs = checkNAs,  
                    varonly = varonly, nresample = nresample,  
                    standardise = standardise, tol = tol)  
  else if (!is.null(ix) && !is.null(iy))  
    .LinStatExpCov2d(X = X, Y = Y, ix = ix, iy = iy,  
                    weights = weights, subset = subset,  
                    block = block, checkNAs = checkNAs,  
                    varonly = varonly, nresample = nresample,  
                    standardise = standardise, tol = tol)  
  else  
    stop("incorrect call to ", sQuote("LinStatExpCov()"))  
}  
◇
```

Fragment referenced in [3a](#).

Uses: block [26f](#), [27b](#), subset [26ade](#), weights [25b](#), weights, [25cd](#).

### 2.1.1 One-Dimensional Case (“1d”)

We assume that  $\mathbf{x}_i$  and  $\mathbf{y}_i$  for  $i = 1, \dots, N$  are available as numeric matrices  $\mathbf{X}$  and  $\mathbf{Y}$  with  $N$  rows as well as  $P$  and  $Q$  columns, respectively. The special case of a dummy matrix  $\mathbf{X}$  with  $P$  columns can also be represented by a factor at  $P$  levels. The vector of case weights `weights` can be stored as `integer` or `double` (possibly resulting from an aggregation of  $N > \text{INT\_MAX}$  observations). The subset vector `subset` may contain the elements  $1, \dots, N$  as `integer` or `double` (for  $N > \text{INT\_MAX}$ ) and can be longer than  $N$ . The `subset` vector MUST be sorted. `block` is a factor at  $B$  levels of length  $N$ .

*< Check weights, subset, block 5a > ≡*

```
if (is.null(weights)) weights <- integer(0)

if (length(weights) > 0) {
  if (!(N == length(weights)) && all(weights >= 0))
    stop("incorrect weights")
  if (checkNAs) stopifnot(!anyNA(weights))
}

if (is.null(subset)) subset <- integer(0)

if (length(subset) > 0 && checkNAs) {
  rs <- range(subset)
  if (anyNA(rs)) stop("no missing values allowed in subset")
  if (!(rs[2] <= N) && (rs[1] >= 1L))
    stop("incorrect subset")
}

if (is.null(block)) block <- integer(0)

if (length(block) > 0) {
  if (!(N == length(block)) && is.factor(block))
    stop("incorrect block")
  if (checkNAs) stopifnot(!anyNA(block))
}
◇
```

Fragment referenced in [6](#), [8](#), [15b](#).

Uses: [block 26f](#), [27b](#), [N 23bc](#), [subset 26ade](#), [weights 25b](#).

Missing values are only allowed in *X* and *Y*, all other vectors must not contain NAs. Missing values are dealt with by excluding the corresponding observations from the subset vector.

*< Handle Missing Values 5b > ≡*

```
ms <- !complete.cases(X, Y)
if (all(ms))
  stop("all observations are missing")
if (any(ms)) {
  if (length(subset) > 0) {
    if (all(subset %in% which(ms)))
      stop("all observations are missing")
    subset <- subset[!(subset %in% which(ms))]
  } else {
    subset <- seq_len(N)[-which(ms)]
  }
}
◇
```

Fragment referenced in [6](#).

Uses: [N 23bc](#), [subset 26ade](#).

The logical argument `varonly` triggers the computation of the diagonal elements of the covariance matrix  $\Sigma$  only. `nresample` permuted linear statistics under the null hypothesis  $H_0$  are returned on the original and standardised scale (the latter only when `standardise` is `TRUE`). Variances smaller than `tol` are treated as being zero.

`<LinStatExpCov1d 6> ≡`

```
.LinStatExpCov1d <-  
function(X, Y, weights = integer(0), subset = integer(0), block = integer(0),  
        checkNAs = TRUE, varonly = FALSE, nresample = 0, standardise = FALSE,  
        tol = sqrt(.Machine$double.eps))  
{  
  if (NROW(X) != NROW(Y))  
    stop("dimensions of X and Y don't match")  
  N <- NROW(X)  
  
  if (is.integer(X)) {  
    if (is.null(attr(X, "levels")) || checkNAs) {  
      rg <- range(X)  
      if (anyNA(rg))  
        stop("no missing values allowed in X")  
      stopifnot(rg[1] > 0) # no missing values allowed here!  
      if (is.null(attr(X, "levels")))  
        attr(X, "levels") <- seq_len(rg[2])  
    }  
  }  
  
  if (is.factor(X) && checkNAs)  
    stopifnot(!anyNA(X))  
  
  < Check weights, subset, block 5a >  
  
  if (checkNAs) {  
    < Handle Missing Values 5b >  
  }  
  
  ret <- .Call(R_ExpectationCovarianceStatistic, X, Y, weights, subset,  
             block, as.integer(varonly), as.double(tol))  
  ret$varonly <- as.logical(ret$varonly)  
  ret$Xfactor <- as.logical(ret$Xfactor)  
  if (nresample > 0) {  
    ret$PermutedLinearStatistic <-  
      .Call(R_PermutedLinearStatistic, X, Y, weights, subset,  
           block, as.double(nresample))  
    if (standardise)  
      ret$StandardisedPermutedLinearStatistic <-  
        .Call(R_StandardisePermutedLinearStatistic, ret)  
  }  
  class(ret) <- c("LinStatExpCov1d", "LinStatExpCov")  
  ret  
}  
◇
```

Fragment referenced in [3a](#).

Uses: [block 26f](#), [27b](#), [N 23bc](#), [NROW 130b](#), [R\\_ExpectationCovarianceStatistic 31a](#), [R\\_PermutedLinearStatistic 37](#),  
[subset 26ade](#), [weights 25b](#), [weights, 25cd](#).

Here is a simple example. We have five groups and a uniform outcome (rounded to one digit) and want to test independence of group membership and outcome. The simplest way is to set-up the dummy matrix explicitly:

```
> isequal <-  
+ function(a, b) {  
+   attributes(a) <- NULL  
+   attributes(b) <- NULL  
+   if (!isTRUE(all.equal(a, b))) {  
+     print(a, digits = 10)  
+     print(b, digits = 10)  
+   }  
+ }
```

```

+         FALSE
+     } else
+         TRUE
+ }
> library("libcoin")
> set.seed(290875)
> x <- gl(5, 20)
> y <- round(runif(length(x)), 1)
> ls1 <- LinStatExpCov(X = model.matrix(~ x - 1), Y = matrix(y, ncol = 1))
> ls1$LinearStatistic

[1] 8.8 9.5 10.3 9.8 10.5

> tapply(y, x, sum)

  1  2  3  4  5
8.8 9.5 10.3 9.8 10.5

```

The linear statistic is simply the sum of the response in each group. Alternatively, we can compute the same object without setting-up the dummy matrix:

```

> ls2 <- LinStatExpCov(X = x, Y = matrix(y, ncol = 1))
> all.equal(ls1[-grep("Xfactor", names(ls1))],
+          ls2[-grep("Xfactor", names(ls2))])

[1] TRUE

```

The results are identical, except for a logical indicating that a factor was used to represent the dummy matrix  $X$ .

## 2.1.2 Two-Dimensional Case (“2d”)

Sometimes the data takes only a few unique values and considerable computational speedups can be achieved taking this information into account. Let  $\mathbf{ix}$  denote an integer vector with elements  $0, \dots, L_x$  of length  $N$  and  $\mathbf{iy}$  an integer vector with elements  $0, \dots, L_y$ , also of length  $N$ . The matrix  $X$  is now of dimension  $(L_x + 1) \times P$  and the matrix  $Y$  of dimension  $(L_y + 1) \times Q$ . The combination of  $X$  and  $\mathbf{ix}$  means that the  $i$ th observation corresponds to the row  $X[\mathbf{ix}[i] + 1, ]$ . This looks cumbersome in R notation but is a very efficient way of dealing with missing values at C level. By convention, elements of  $\mathbf{ix}$  being zero denote a missing value (NAs are not allowed in  $\mathbf{ix}$  and  $\mathbf{iy}$ ). Thus, the first row of  $X$  corresponds to a missing value. If the first row is simply zero, missing values do not contribute to any of the sums computed later. Even more important is the fact that all entities, such as linear statistics etc., can be computed from the two-way tabulation (therefore the abbreviation “2d”) over the  $N$  elements of  $\mathbf{ix}$  and  $\mathbf{iy}$ . Once such a table was computed, the remaining computations can be performed in dimension  $L_x \times L_y$ , typically much smaller than  $N$ .

*< LinStatExpCov2d 8 >* ≡

```
.LinStatExpCov2d <-  
function(X = numeric(0), Y, ix, iy, weights = integer(0), subset = integer(0),  
        block = integer(0), checkNAs = TRUE, varonly = FALSE, nresample = 0,  
        standardise = FALSE, tol = sqrt(.Machine$double.eps))  
{  
  IF <- function(x) is.integer(x) || is.factor(x)  
  
  if (!(length(ix) == length(iy)) && IF(ix) && IF(iy))  
    stop("incorrect ix and/or iy")  
  N <- length(ix)  
  
  < Check ix 9a >  
  
  < Check iy 9b >  
  
  if (length(X) > 0) {  
    if (!(NROW(X) == (length(attr(ix, "levels")) + 1) &&  
        all(complete.cases(X))))  
      stop("incorrect X")  
  }  
  
  if (!(NROW(Y) == (length(attr(iy, "levels")) + 1) &&  
      all(complete.cases(Y))))  
    stop("incorrect Y")  
  
  < Check weights, subset, block 5a >  
  
  ret <- .Call(R_ExpectationCovarianceStatistic_2d, X, ix, Y, iy,  
             weights, subset, block, as.integer(varonly), as.double(tol))  
  ret$varonly <- as.logical(ret$varonly)  
  ret$Xfactor <- as.logical(ret$Xfactor)  
  if (nresample > 0) {  
    ret$PermutedLinearStatistic <-  
      .Call(R_PermutedLinearStatistic_2d, X, ix, Y, iy, block, nresample, ret$Table)  
    if (standardise)  
      ret$StandardisedPermutedLinearStatistic <-  
        .Call(R_StandardisePermutedLinearStatistic, ret)  
  }  
  class(ret) <- c("LinStatExpCov2d", "LinStatExpCov")  
  ret  
}  
◇
```

Fragment referenced in [3a](#).

Uses: [block 26f](#), [27b](#), [N 23bc](#), [NROW 130b](#), [R\\_ExpectationCovarianceStatistic\\_2d 41a](#), [R\\_PermutedLinearStatistic\\_2d 48](#),  
[subset 26ade](#), [weights 25b](#), [weights, 25cd](#), [x 23d](#), [24bc](#).

*ix* and *iy* can be factors but without any missing values

⟨ *Check ix 9a* ⟩ ≡

```
if (is.null(attr(ix, "levels"))) {
  rg <- range(ix)
  if (anyNA(rg))
    stop("no missing values allowed in ix")
  stopifnot(rg[1] >= 0)
  attr(ix, "levels") <- seq_len(rg[2])
} else {
  ## lev can be data.frame (see inum::inum)
  lev <- attr(ix, "levels")
  if (!is.vector(lev)) lev <- seq_len(NROW(lev))
  attr(ix, "levels") <- lev
  if (checkNAs) stopifnot(!anyNA(ix))
}
◇
```

Fragment referenced in 8, 15b.  
Uses: NROW 130b.

⟨ *Check iy 9b* ⟩ ≡

```
if (is.null(attr(iy, "levels"))) {
  rg <- range(iy)
  if (anyNA(rg))
    stop("no missing values allowed in iy")
  stopifnot(rg[1] >= 0)
  attr(iy, "levels") <- seq_len(rg[2])
} else {
  ## lev can be data.frame (see inum::inum)
  lev <- attr(iy, "levels")
  if (!is.vector(lev)) lev <- seq_len(NROW(lev))
  attr(iy, "levels") <- lev
  if (checkNAs) stopifnot(!anyNA(iy))
}
◇
```

Fragment referenced in 8, 15b.  
Uses: NROW 130b.

In our small example, we can set-up the data in the following way

```
> X <- rbind(0, diag(nlevels(x)))
> ix <- unclass(x)
> ylev <- sort(unique(y))
> Y <- rbind(0, matrix(ylev, ncol = 1))
> iy <- .bincode(y, breaks = c(-Inf, ylev, Inf))
> ls3 <- LinStatExpCov(X = X, ix = ix, Y = Y, iy = iy)
> all.equal(ls1[-grep("Table", names(ls1))],
+          ls3[-grep("Table", names(ls3))])

[1] TRUE

> ### works also with factors
> ls3 <- LinStatExpCov(X = X, ix = factor(ix), Y = Y, iy = factor(iy))
> all.equal(ls1[-grep("Table", names(ls1))],
+          ls3[-grep("Table", names(ls3))])

[1] TRUE
```

Similar to the one-dimensional case, we can also omit the X matrix here

```
> ls4 <- LinStatExpCov(ix = ix, Y = Y, iy = iy)
> all.equal(ls3[-grep("Xfactor", names(ls3))],
+          ls4[-grep("Xfactor", names(ls4))])
```

```
[1] TRUE
```

It is important to note that all computations are based on the tabulations

```
> ls3$Table
```

```
, , 1
```

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]	[,9]	[,10]	[,11]	[,12]
[1,]	0	0	0	0	0	0	0	0	0	0	0	0
[2,]	0	0	4	4	1	2	3	0	1	2	3	0
[3,]	0	2	2	1	2	2	5	0	1	1	3	1
[4,]	0	1	1	4	0	1	5	2	0	2	3	1
[5,]	0	0	2	2	4	2	2	1	3	2	1	1
[6,]	0	1	3	1	1	1	2	2	2	6	1	0

```
> xtabs(~ ix + iy)
```

	iy										
ix	1	2	3	4	5	6	7	8	9	10	11
1	0	4	4	1	2	3	0	1	2	3	0
2	2	2	1	2	2	5	0	1	1	3	1
3	1	1	4	0	1	5	2	0	2	3	1
4	0	2	2	4	2	2	1	3	2	1	1
5	1	3	1	1	1	2	2	6	1	0	

where the former would record missing values in the first row / column.

### 2.1.3 Methods and Tests

Objects of class "LinStatExpCov" returned by `LinStatExpCov()` contain the symmetric covariance matrix as a vector of the lower triangular elements. The `vcov` method allows to extract the full covariance matrix from such an object.

```
<vcov LinStatExpCov 10> ≡
```

```
vcov.LinStatExpCov <-
function(object, ...)
{
  if (object$varonly)
    stop("cannot extract covariance matrix")
  drop(.Call(R_unpack_sym, object$Covariance, NULL, 0L))
}
◇
```

Fragment referenced in [3a](#).

Uses: `R_unpack_sym` [139](#).

```
> ls1$Covariance
```

```
[1] 1.3572364 -0.3393091 -0.3393091 -0.3393091 -0.3393091 1.3572364
[7] -0.3393091 -0.3393091 -0.3393091 1.3572364 -0.3393091 -0.3393091
[13] 1.3572364 -0.3393091 1.3572364
```

```
> vcov(ls1)
```

```

      [,1]      [,2]      [,3]      [,4]      [,5]
[1,]  1.3572364 -0.3393091 -0.3393091 -0.3393091 -0.3393091
[2,] -0.3393091  1.3572364 -0.3393091 -0.3393091 -0.3393091
[3,] -0.3393091 -0.3393091  1.3572364 -0.3393091 -0.3393091
[4,] -0.3393091 -0.3393091 -0.3393091  1.3572364 -0.3393091
[5,] -0.3393091 -0.3393091 -0.3393091 -0.3393091  1.3572364

```

The most important task is, however, to compute test statistics and  $p$ -values. `doTest()` allows to compute the statistics  $c_{\max}$  (taking `alternative` into account) and  $c_{\text{quad}}$  along with the corresponding  $p$ -values. If `nresample = 0` was used in the call to `LinStatExpCov()`,  $p$ -values are obtained from the limiting asymptotic distribution whenever such a thing is available at reasonable costs. Otherwise, the permutation  $p$ -value is returned (along with the permuted test statistics when `PermutedStatistics` is `TRUE`). The  $p$ -values (`lower = FALSE`) or  $(1-p)$ -values (`lower = TRUE`) can be computed on the log-scale.

```

⟨ doTest Prototype 11 ⟩ ≡
  (object, teststat = c("maximum", "quadratic", "scalar"),
   alternative = c("two.sided", "less", "greater"), pvalue = TRUE,
   lower = FALSE, log = FALSE, PermutedStatistics = FALSE,
   minbucket = 10L, ordered = TRUE, maxselect = object$Xfactor,
   pargs = GenzBretz())◇

```

Fragment referenced in [12](#), [18](#).

`< doTest 12 > ≡`

```
### note: lower = FALSE => p-value; lower = TRUE => 1 - p-value
doTest <-
function(doTest Prototype 11)
{
  teststat <- match.arg(teststat, choices = c("maximum", "quadratic", "scalar"))
  if (!any(teststat == c("maximum", "quadratic", "scalar")))
    stop("incorrect teststat")
  alternative <- alternative[1]
  if (!any(alternative == c("two.sided", "less", "greater")))
    stop("incorrect alternative")

  if (maxselect)
    stopifnot(object$Xfactor)

  if (teststat == "quadratic" || maxselect) {
    if (alternative != "two.sided")
      stop("incorrect alternative")
  }

  test <- which(c("maximum", "quadratic", "scalar") == teststat)
  if (test == 3) {
    if (length(object$LinearStatistic) != 1)
      stop("scalar test statistic not applicable")
    test <- 1L # scalar is maximum internally
  }
  alt <- which(c("two.sided", "less", "greater") == alternative)

  if (!pvalue && (NCOL(object$PermutedLinearStatistic) > 0))
    object$PermutedLinearStatistic <- matrix(NA_real_, nrow = 0, ncol = 0)

  if (!maxselect) {
    if (teststat == "quadratic") {
      ret <- .Call(R_QuadraticTest, object, as.integer(pvalue), as.integer(lower),
                  as.integer(log), as.integer(PermutedStatistics))
    } else {
      ret <- .Call(R_MaximumTest, object, as.integer(alt), as.integer(pvalue),
                  as.integer(lower), as.integer(log), as.integer(PermutedStatistics),
                  as.integer(pargs$maxpts), as.double(pargs$releps),
                  as.double(pargs$abseps))
      if (teststat == "scalar") {
        var <- if (object$varonly) object$Variance else object$Covariance
        ret$TestStatistic <- object$LinearStatistic - object$Expectation
        ret$TestStatistic <-
          if (var > object$tol) ret$TestStatistic / sqrt(var) else NaN
      }
    }
  } else {
    ret <- .Call(R_MaximallySelectedTest, object, as.integer(ordered), as.integer(test),
                as.integer(minbucket), as.integer(lower), as.integer(log))
  }
  if (!PermutedStatistics) ret$PermutedStatistics <- NULL
  ret
}
◇
```

Fragment referenced in [3a](#).  
Uses: [NCOL 130c](#).

```
> ### c_max test statistic
> ### no p-value
> doTest(ls1, teststat = "maximum", pvalue = FALSE)
```

```

$TestStatistic
[1] 0.8411982

$p.value
[1] NA

> ### p-value
> doTest(ls1, teststat = "maximum")

$TestStatistic
[1] 0.8411982

$p.value
[1] 0.8852087

> ### log(p)-value
> doTest(ls1, teststat = "maximum", log = TRUE)

$TestStatistic
[1] 0.8411982

$p.value
[1] 0.108822

> ### (1-p)-value
> doTest(ls1, teststat = "maximum", lower = TRUE)

$TestStatistic
[1] 0.8411982

$p.value
[1] 0.1150168

> ### log(1 - p)-value
> doTest(ls1, teststat = "maximum", lower = TRUE, log = TRUE)

$TestStatistic
[1] 0.8411982

$p.value
[1] -2.164164

> ### quadratic
> doTest(ls1, teststat = "quadratic")

$TestStatistic
[1] 1.077484

$p.value
[1] 0.897828

```

Sometimes we are interested in contrasts of linear statistics and their corresponding properties. Examples include linear-by-linear association tests, where we assign numeric scores to each level of a factor. To implement this, we implement `lmult()` so that we can then left-multiply a matrix to an object of class `"LinStatExpCov"`.

*< Contrasts 14 >* ≡

```
lmult <-  
function(x, object)  
{  
  stopifnot(!object$varonly)  
  stopifnot(is.numeric(x))  
  if (is.vector(x)) x <- matrix(x, nrow = 1)  
  P <- object$dimension[1]  
  stopifnot(ncol(x) == P)  
  Q <- object$dimension[2]  
  ret <- object  
  xLS <- x %*% matrix(object$LinearStatistic, nrow = P)  
  xExp <- x %*% matrix(object$Expectation, nrow = P)  
  xExpX <- x %*% matrix(object$ExpectationX, nrow = P)  
  if (Q == 1) {  
    xCov <- tcrossprod(x %*% vcov(object), x)  
  } else {  
    zmat <- matrix(0, nrow = P * Q, ncol = nrow(x))  
    mat <- rbind(t(x), zmat)  
    mat <- mat[rep.int(seq_len(nrow(mat)), Q - 1),, drop = FALSE]  
    mat <- rbind(mat, t(x))  
    mat <- matrix(mat, ncol = Q * nrow(x))  
    mat <- t(mat)  
    xCov <- tcrossprod(mat %*% vcov(object), mat)  
  }  
  if (!is.matrix(xCov)) xCov <- matrix(xCov)  
  if (length(object$PermutedLinearStatistic) > 0) {  
    xPS <- apply(object$PermutedLinearStatistic, 2, function(y)  
      as.vector(x %*% matrix(y, nrow = P)))  
    if (!is.matrix(xPS)) xPS <- matrix(xPS, nrow = 1)  
    ret$PermutedLinearStatistic <- xPS  
  }  
  ret$LinearStatistic <- as.vector(xLS)  
  ret$Expectation <- as.vector(xExp)  
  ret$ExpectationX <- as.vector(xExpX)  
  ret$Covariance <- as.vector(xCov[lower.tri(xCov, diag = TRUE)])  
  ret$Variance <- diag(xCov)  
  ret$dimension <- c(NROW(x), Q)  
  ret$Xfactor <- FALSE  
  if (length(object$StandardisedPermutedLinearStatistic) > 0)  
    ret$StandardisedPermutedLinearStatistic <-  
      .Call(R_StandardisePermutedLinearStatistic, ret)  
  ret  
}  
◇
```

Fragment referenced in 3a.

Uses: NROW 130b, P 24a, Q 24e, x 23d, 24bc, y 24df, 25a.

Here is an example for a linear-by-linear association test.

```
> set.seed(29)  
> ls1d <- LinStatExpCov(X = model.matrix(~ x - 1), Y = matrix(y, ncol = 1),  
+                       nresample = 10, standardise = TRUE)  
> set.seed(29)  
> ls1s <- LinStatExpCov(X = as.double(1:5)[x], Y = matrix(y, ncol = 1),  
+                       nresample = 10, standardise = TRUE)  
> ls1c <- lmult(1:5, ls1d)  
> stopifnot(isequal(ls1c, ls1s))  
> set.seed(29)  
> ls1d <- LinStatExpCov(X = model.matrix(~ x - 1), Y = matrix(c(y, y), ncol = 2),  
+                       nresample = 10, standardise = TRUE)
```

```

> set.seed(29)
> ls1s <- LinStatExpCov(X = as.double(1:5)[x], Y = matrix(c(y, y), ncol = 2),
+                       nresample = 10, standardise = TRUE)
> ls1c <- lmult(1:5, ls1d)
> stopifnot(isequal(ls1c, ls1s))

```

## 2.1.4 Tabulations

The tabulation of `ix` and `iy` can be computed without necessarily computing the corresponding linear statistics via `ctabs()`.

```

<ctabs Prototype 15a> ≡
  (ix, iy = integer(0), block = integer(0), weights = integer(0),
   subset = integer(0), checkNAs = TRUE)◊

```

Fragment referenced in [15b](#), [19](#).

Uses: block [26f](#), [27b](#), subset [26ade](#), weights [25b](#).

"ctabs.R" 15b≡

```

<R Header 154a>
ctabs <-
function(<ctabs Prototype 15a>
{
  stopifnot(is.integer(ix) || is.factor(ix))
  N <- length(ix)

  <Check ix 9a>

  if (length(iy) > 0) {
    stopifnot(length(iy) == N)
    stopifnot(is.integer(iy) || is.factor(iy))
    <Check iy 9b>
  }

  <Check weights, subset, block 5a>

  if (length(iy) == 0)
    if (length(block) == 0)
      .Call(R_OneTableSums, ix, weights, subset)
    else
      .Call(R_TwoTableSums, ix, block, weights, subset)[, -1, drop = FALSE]
  else if (length(block) == 0)
    .Call(R_TwoTableSums, ix, iy, weights, subset)
  else
    .Call(R_ThreeTableSums, ix, iy, block, weights, subset)
}
◊

```

Uses: block [26f](#), [27b](#), N [23bc](#), R\_OneTableSums [111a](#), R\_ThreeTableSums [119b](#), R\_TwoTableSums [115a](#), subset [26ade](#), weights [25b](#), weights, [25cd](#).

```

> t1 <- ctab(ix = ix, iy = iy)
> t2 <- xtab(~ ix + iy)
> max(abs(t1[-1, -1] - t2))

```

```
[1] 0
```



## 2.2 Manual Pages

"LinStatExpCov.Rd" 17

```
\name{LinStatExpCov}
\alias{LinStatExpCov}
\alias{lmult}
\title{
  Linear Statistics with Expectation and Covariance
}
\description{
  Strasser-Weber type linear statistics and their expectation
  and covariance under the independence hypothesis
}
\usage{
LinStatExpCov(LinStatExpCov Prototype 3b)
lmult(x, object)
}
\arguments{
  \item{X}{numeric matrix of transformations.}
  \item{Y}{numeric matrix of influence functions.}
  \item{ix}{an optional integer vector expanding X.}
  \item{iy}{an optional integer vector expanding Y.}
  \item{weights}{an optional integer vector of non-negative case weights.}
  \item{subset}{an optional integer vector defining a subset of observations.}
  \item{block}{an optional factor defining independent blocks of observations.}
  \item{checkNAs}{a logical for switching off missing value checks. This
    included switching off checks for suitable values of subset.
    Use at your own risk.}
  \item{varonly}{a logical asking for variances only.}
  \item{nresample}{an integer defining the number of permuted statistics to draw.}
  \item{standardise}{a logical asking to standardise the permuted statistics.}
  \item{tol}{tolerance for zero variances.}
  \item{x}{a contrast matrix to be left-multiplied in case X was a factor.}
  \item{object}{an object of class "LinStatExpCov".}
}
\details{
  The function, after minimal preprocessing, calls the underlying C code
  and computes the linear statistic, its expectation and covariance and,
  optionally, nresample samples from its permutation distribution.

  When both ix and iy are missing, the number of rows of
  X and Y is the same, ie the number of observations.

  When X is missing and ix a factor, the code proceeds as
  if X were a dummy matrix of ix without explicitly
  computing this matrix.

  Both ix and iy being present means the code treats them
  as subsetting vectors for X and Y. Note that ix = 0
  or iy = 0 means that the corresponding observation is missing
  and the first row of X and Y must be zero.

  lmult allows left-multiplication of a contrast matrix when X
  was (equivalent to) a factor.
}
\value{
  A list.
}
\references{
  Strasser, H. and Weber, C. (1999). On the asymptotic theory of permutation
  statistics. Mathematical Methods of Statistics 8(2), 220--250.
}
\examples{
wilcox.test(Ozone ~ Month, data = airquality, subset = Month \%in\% c(5, 8),
            exact = FALSE, correct = FALSE)

aq <- subset(airquality, Month \%in\% c(5, 8))
X <- as.double(aq$Month == 5)
Y <- as.double(rank(aq$Ozone, na.last = "keep"))
d-Test(LinStatExpCov(X, Y))
}
```

"doTest.Rd" 18≡

```
\name{doTest}
\alias{doTest}
\title{
  Permutation Test
}
\description{
  Perform permutation test for a linear statistic
}
\usage{
doTest(doTest Prototype 11)
}
\arguments{
\item{object}{an object returned by \link{LinStatExpCov}.}
\item{teststat}{type of test statistic to use.}
\item{alternative}{alternative for scalar or maximum-type statistics.}
\item{pvalue}{a logical indicating if a p-value shall be computed.}
\item{lower}{a logical indicating if a p-value (lower is FALSE)
  or 1 - p-value (lower is TRUE) shall be returned.}
\item{log}{a logical, if TRUE probabilities are log-probabilities.}
\item{PermutedStatistics}{a logical, return permuted test statistics.}
\item{minbucket}{minimum weight in either of two groups for maximally selected
  statistics.}
\item{ordered}{a logical, if TRUE maximally selected statistics assume
  that the cutpoints are ordered.}
\item{maxselect}{a logical, if TRUE maximally selected statistics are
  computed. This requires that X was an implicitly defined design
  matrix in \link{LinStatExpCov}.}
\item{pargs}{arguments as in \link[mvtnorm:algorithms]{GenzBretz}.}
}
\details{
  Computes a test statistic, a corresponding p-value and, optionally, cutpoints
  for maximally selected statistics.
}
\value{
  A list.
}
\keyword{htest}
◇
```

"ctabs.Rd" 19≡

```
\name{ctabs}
\alias{ctabs}
\title{
  Cross Tabulation
}
\description{
  Efficient weighted cross tabulation of two factors and a block
}
\usage{
ctabs(ctabs Prototype 15a)
}
\arguments{
  \item{ix}{a integer of positive values with zero indicating a missing.}
  \item{iy}{an optional integer of positive values with zero indicating a
    missing.}
  \item{block}{an optional blocking factor without missings.}
  \item{weights}{an optional vector of case weights, integer or double.}
  \item{subset}{an optional integer vector indicating a subset.}
  \item{checkNAs}{a logical for switching off missing value checks.}
}
\details{
  A faster version of xtabs(weights ~ ix + iy + block, subset).
}
\value{
  If block is present, a three-way table. Otherwise,
  a one- or two-dimensional table.
}
\examples{
ctabs(ix = 1:5, iy = 1:5, weights = 1:5 / 5)
}
\keyword{univar}
◇
```

Uses: [block 26f](#), [27b](#), [subset 26ade](#), [weights 25b](#), [weights, 25cd](#).

# Chapter 3

## C Code

The main motivation to implement the **libcoin** package comes from the demand to compute high-dimensional linear statistics (with large  $P$  and  $Q$ ) and the corresponding test statistics very often, either for sampling from the permutation null distribution  $H_0$  or for different subsets of the data. Especially the latter task can be performed *without* actually subsetting the data via the **subset** argument very efficiently (in terms of memory consumption and, depending on the circumstances, speed).

We start with the definition of some macros and global variables in the header files.

### 3.1 Header and Source Files

"libcoin\_internal.h" 20a≡

```
< C Header 154b >
< R Includes 20b >
< C Macros 21a >
< C Global Variables 21b >
◇
```

These includes provide some R infrastructure at C level.

< R Includes 20b > ≡

```
#define STRICT_R_HEADERS
#define USE_FC_LEN_T
#include <float.h> /* for DBL_MIN */
#include <R.h>
#include <Rinternals.h>
#include <Rversion.h> /* for R_VERSION */
#include <R_ext/Lapack.h> /* for dspev */
#ifndef FCONE
# define FCONE
#endif
◇
```

Fragment referenced in 20a.

We need three macros: **S** computes the element  $\sigma_{ij}$  of a symmetric  $n \times n$  matrix when only the lower triangular elements are stored. **LE** implements  $\leq$  with some tolerance, **GE** implements  $\geq$ .

*< C Macros 21a >* ≡

```
#define S(i, j, n) ((i) >= (j) ? (n) * (j) + (i) - (j) * ((j) + 1) / 2 : (n) * (i) + (j) - (i) * ((i) + 1)
#define LE(x, y, tol) ((x) < (y)) || (fabs((x) - (y)) < (tol))
#define GE(x, y, tol) ((x) > (y)) || (fabs((x) - (y)) < (tol))
◇
```

Fragment referenced in 20a.

Defines: GE 51, 54, LE 54, S 35b, 36a, 44, 45, 57b, 58b, 59b, 62, 64a, 68, 69a, 73a, 76b, 88a, 99, 134, 135a, 137, 143b.

Uses: x 23d, 24bc, y 24df, 25a.

*< C Global Variables 21b >* ≡

```
#define ALTERNATIVE_twosided 1
#define ALTERNATIVE_less 2
#define ALTERNATIVE_greater 3

#define TESTSTAT_maximum 1
#define TESTSTAT_quadratic 2

#define LinearStatistic_SLOT 0
#define Expectation_SLOT 1
#define Covariance_SLOT 2
#define Variance_SLOT 3
#define ExpectationX_SLOT 4
#define varonly_SLOT 5
#define dim_SLOT 6
#define ExpectationInfluence_SLOT 7
#define CovarianceInfluence_SLOT 8
#define VarianceInfluence_SLOT 9
#define Xfactor_SLOT 10
#define tol_SLOT 11
#define PermutedLinearStatistic_SLOT 12
#define StandardisedPermutedLinearStatistic_SLOT 13
#define TableBlock_SLOT 14
#define Sumweights_SLOT 15
#define Table_SLOT 16

#define DoSymmetric 1
#define DoCenter 1
#define DoVarOnly 1
#define Power1 1
#define Power2 2
#define Offset0 0
◇
```

Fragment referenced in 20a.

Defines: CovarianceInfluence\_SLOT 144c, 147b, 148, Covariance\_SLOT 143bc, 147b, 148, dim\_SLOT 141c, 142a, 147b, 148, DoCenter 77d, 81c, 84a, 85b, 88a, 94b, 106c, DoSymmetric 77d, 84a, 88a, DoVarOnly 35bcd, 44, ExpectationInfluence\_SLOT 144b, 147b, 148, ExpectationX\_SLOT 144a, 147b, 148, Expectation\_SLOT 143a, 147b, 148, LinearStatistic\_SLOT 142d, 147b, 148, Offset0 33b, 34a, 37, 41a, 43b, 44, 81a, 83a, 84c, 87a, 90a, 94b, 102b, 106c, 111a, 115a, 119b, 123c, 127a, PermutedLinearStatistic\_SLOT 146bc, 147b, 148, Power1 81c, 85b, 106c, Power2 84a, 88a, StandardisedPermutedLinearStatistic\_SLOT 147b, 148, Sumweights\_SLOT 145b, 146a, 147b, 148, 149b, TableBlock\_SLOT 34a, 145a, 146a, 147b, 148, 149b, Table\_SLOT 145cd, 147b, 148, 150, tol\_SLOT 146d, 147b, 148, VarianceInfluence\_SLOT 144d, 147b, 148, Variance\_SLOT 143b, 147b, 148, varonly\_SLOT 142b, 147b, 148, Xfactor\_SLOT 142c, 147b, 148.

The corresponding header file contains definitions of functions that can be called via `.Call()` from the **libcoin** package. In addition, packages linking to **libcoin** can access these function at C level (at your own risk, of course!).

"libcoin.h" 22a≡

```
⟨ C Header 154b ⟩
#include "libcoin_internal.h"
⟨ Function Prototypes 22b ⟩
◇
```

⟨ Function Prototypes 22b ⟩ ≡

```
extern ⟨ R_ExpectationCovarianceStatistic Prototype 30c ⟩;
extern ⟨ R_PermutedLinearStatistic Prototype 36b ⟩;
extern ⟨ R_StandardisePermutedLinearStatistic Prototype 38b ⟩;
extern ⟨ R_ExpectationCovarianceStatistic_2d Prototype 40a ⟩;
extern ⟨ R_PermutedLinearStatistic_2d Prototype 47a ⟩;
extern ⟨ R_QuadraticTest Prototype 50b ⟩;
extern ⟨ R_MaximumTest Prototype 52b ⟩;
extern ⟨ R_MaximallySelectedTest Prototype 55a ⟩;
extern ⟨ R_ExpectationInfluence Prototype 80b ⟩;
extern ⟨ R_CovarianceInfluence Prototype 82 ⟩;
extern ⟨ R_ExpectationX Prototype 84b ⟩;
extern ⟨ R_CovarianceX Prototype 86 ⟩;
extern ⟨ R_Sums Prototype 89c ⟩;
extern ⟨ R_KronSums Prototype 94a ⟩;
extern ⟨ R_KronSums_Permutation Prototype 102a ⟩;
extern ⟨ R_colSums Prototype 106b ⟩;
extern ⟨ R_OneTableSums Prototype 110b ⟩;
extern ⟨ R_TwoTableSums Prototype 114b ⟩;
extern ⟨ R_ThreeTableSums Prototype 119a ⟩;
extern ⟨ R_order_subset_wrt_block Prototype 123b ⟩;
extern ⟨ R_quadform Prototype 60b ⟩;
extern ⟨ R_kronecker Prototype 132b ⟩;
extern ⟨ R_MPinv_sym Prototype 135b ⟩;
extern ⟨ R_unpack_sym Prototype 138a ⟩;
extern ⟨ R_pack_sym Prototype 140a ⟩;
◇
```

Fragment referenced in [22a](#).

The C file `libcoin.c` contains all C functions and corresponding R interfaces.

"libcoin.c" 22c≡

```
⟨ C Header 154b ⟩
#include "libcoin_internal.h"
#include <R_ext/stats_stubs.h> /* for S_rcont2 */
#include <mvtnormAPI.h> /* for calling mvtnorm */
⟨ Function Definitions 23a ⟩
◇
```

*Function Definitions 23a*  $\equiv$

*MoreUtils 130a*  
*Memory 141a*  
*P-Values 64b*  
*KronSums 93b*  
*colSums 106a*  
*SimpleSums 89b*  
*Tables 110a*  
*Utils 123a*  
*LinearStatistics 77b*  
*Permutations 127b*  
*ExpectationCovariances 78a*  
*Test Statistics 57a*  
*User Interface 30a*  
*2d User Interface 39b*  
*Tests 50a*

◇

Fragment referenced in [22c](#).

## 3.2 Variables

$N$  is the number of observations

*R N Input 23b*  $\equiv$

SEXP  $N$ ,

◇

Fragment referenced in [89c](#).

Defines:  $N$  [5ab](#), [6](#), [8](#), [15b](#), [23c](#), [33ab](#), [34ab](#), [35abcd](#), [37](#), [41a](#), [67](#), [77d](#), [81ac](#), [83a](#), [84ac](#), [85b](#), [87a](#), [88abc](#), [90a](#), [91a](#), [93a](#), [94b](#), [96b](#), [97a](#), [99](#), [101b](#), [102b](#), [103b](#), [104c](#), [105c](#), [106c](#), [107b](#), [109b](#), [111a](#), [112a](#), [115a](#), [116a](#), [119b](#), [120b](#), [123c](#), [124b](#), [125ab](#), [126a](#), [127a](#), [135a](#).

which at C level is represented as `R_xlen_t` to allow for  $N > \text{INT\_MAX}$

*C integer N Input 23c*  $\equiv$

`R_xlen_t`  $N$

◇

Fragment referenced in [24bc](#), [32](#), [37](#), [41a](#), [77c](#), [81ab](#), [83ab](#), [84c](#), [85a](#), [87ab](#), [90b](#), [91b](#), [92abc](#), [94b](#), [95c](#), [102b](#), [103a](#), [106c](#), [111a](#), [115a](#), [119b](#), [123c](#), [124a](#), [125ab](#), [126b](#).

Defines:  $N$  [5ab](#), [6](#), [8](#), [15b](#), [23b](#), [33ab](#), [34ab](#), [35abcd](#), [37](#), [41a](#), [67](#), [77d](#), [81ac](#), [83a](#), [84ac](#), [85b](#), [87a](#), [88abc](#), [90a](#), [91a](#), [93a](#), [94b](#), [96b](#), [97a](#), [99](#), [101b](#), [102b](#), [103b](#), [104c](#), [105c](#), [106c](#), [107b](#), [109b](#), [111a](#), [112a](#), [115a](#), [116a](#), [119b](#), [120b](#), [123c](#), [124b](#), [125ab](#), [126a](#), [127a](#), [135a](#).

The regressors  $\mathbf{x}_i, i = 1, \dots, N$

*R x Input 23d*  $\equiv$

SEXP  $\mathbf{x}$ ,

◇

Fragment referenced in [30b](#), [39c](#), [47a](#), [77c](#), [84b](#), [85a](#), [86](#), [87b](#), [94a](#), [95c](#), [102a](#), [103a](#), [106b](#), [110b](#), [114b](#), [119a](#).

Defines:  $\mathbf{x}$  [8](#), [14](#), [17](#), [21a](#), [24bc](#), [30d](#), [31ab](#), [33ab](#), [35acd](#), [36c](#), [37](#), [40b](#), [41ab](#), [42a](#), [43b](#), [44](#), [47b](#), [48](#), [77d](#), [84c](#), [85b](#), [87a](#), [88a](#), [94b](#), [95b](#), [96b](#), [97a](#), [99](#), [101b](#), [102b](#), [103b](#), [104c](#), [105c](#), [106c](#), [107b](#), [109b](#), [111a](#), [112a](#), [114a](#), [115a](#), [116a](#), [118b](#), [119b](#), [120b](#), [122b](#), [130bc](#), [131a](#), [135ab](#), [136ab](#), [137](#), [138ab](#), [139](#), [140abc](#).

are either represented as a real matrix with  $N$  rows and  $P$  columns

$\langle C \text{ integer } P \text{ Input } 24a \rangle \equiv$

```
int P
◇
```

Fragment referenced in 24bc, 32, 77c, 78b, 79, 80a, 85a, 87b, 95c, 103a, 149b, 150.

Defines: P 14, 31ab, 33ab, 34a, 35acd, 36a, 37, 41ab, 42a, 43b, 44, 45, 46, 48, 51, 52a, 54, 56, 70, 71, 72, 73a, 74, 75ab, 76ab, 77d, 78b, 79, 80a, 84bc, 85b, 86, 87a, 88a, 94ab, 96b, 97a, 99, 101b, 102ab, 103b, 104c, 105c, 106c, 107b, 109b, 111a, 112a, 114a, 115a, 116a, 118b, 119b, 120b, 122b, 131b, 132a, 135a, 147a, 148.

$\langle C \text{ real } x \text{ Input } 24b \rangle \equiv$

```
double *x,
⟨ C integer N Input 23c ⟩,
⟨ C integer P Input 24a ⟩,
◇
```

Fragment referenced in 95d, 104ab, 107c, 135a.

Defines: x 8, 14, 17, 21a, 23d, 24c, 30d, 31ab, 33ab, 35acd, 36c, 37, 40b, 41ab, 42a, 43b, 44, 47b, 48, 77d, 84c, 85b, 87a, 88a, 94b, 95b, 96b, 97a, 99, 101b, 102b, 103b, 104c, 105c, 106c, 107b, 109b, 111a, 112a, 114a, 115a, 116a, 118b, 119b, 120b, 122b, 130bc, 131a, 135ab, 136ab, 137, 138ab, 139, 140abc.

or as a factor (an integer at C level) at  $P$  levels

$\langle C \text{ integer } x \text{ Input } 24c \rangle \equiv$

```
int *x,
⟨ C integer N Input 23c ⟩,
⟨ C integer P Input 24a ⟩,
◇
```

Fragment referenced in 100a, 105ab, 112b, 116b, 120c.

Defines: x 8, 14, 17, 21a, 23d, 24b, 30d, 31ab, 33ab, 35acd, 36c, 37, 40b, 41ab, 42a, 43b, 44, 47b, 48, 77d, 84c, 85b, 87a, 88a, 94b, 95b, 96b, 97a, 99, 101b, 102b, 103b, 104c, 105c, 106c, 107b, 109b, 111a, 112a, 114a, 115a, 116a, 118b, 119b, 120b, 122b, 130bc, 131a, 135ab, 136ab, 137, 138ab, 139, 140abc.

The influence functions are also either a  $N \times Q$  real matrix

$\langle R \text{ } y \text{ Input } 24d \rangle \equiv$

```
SEXP y,
◇
```

Fragment referenced in 30b, 39c, 47a, 80b, 81b, 82, 83b, 94a, 102a, 114b, 119a, 123b.

Defines: y 14, 21a, 24f, 25a, 30d, 31ab, 33b, 35ab, 36c, 37, 40b, 41ab, 42a, 43b, 44, 47b, 77d, 81ac, 83a, 84a, 94b, 96b, 97a, 99, 101b, 102b, 103b, 104c, 105c, 115a, 116a, 118b, 119b, 120b, 122b, 123c, 133b, 134.

$\langle C \text{ integer } Q \text{ Input } 24e \rangle \equiv$

```
int Q
◇
```

Fragment referenced in 24f, 25a, 32, 78b, 79, 80a, 81ab, 83ab, 94b, 102b, 149b, 150.

Defines: Q 14, 31ab, 33ab, 35abcd, 36a, 37, 41ab, 42a, 43b, 44, 45, 46, 48, 51, 52a, 54, 70, 71, 72, 73abc, 74, 76ab, 77ad, 78b, 79, 80a, 81ac, 83a, 84a, 94b, 96b, 97a, 99, 101b, 102b, 103b, 104c, 105c, 115a, 116a, 118b, 119b, 120b, 122b, 132a, 147a, 148, 149a.

*< C real y Input 24f >* ≡

```
double *y,  
< C integer Q Input 24e >,  
◇
```

Fragment referenced in 77c, 95cd, 100a, 103a, 104ab, 105ab.

Defines: **y** 14, 21a, 24d, 25a, 30d, 31ab, 33b, 35ab, 36c, 37, 40b, 41ab, 42a, 43b, 44, 47b, 77d, 81ac, 83a, 84a, 94b, 96b, 97a, 99, 101b, 102b, 103b, 104c, 105c, 115a, 116a, 118b, 119b, 120b, 122b, 123c, 133b, 134.

or a factor at  $Q$  levels

*< C integer y Input 25a >* ≡

```
int *y,  
< C integer Q Input 24e >,  
◇
```

Fragment referenced in 116b, 120c.

Defines: **y** 14, 21a, 24df, 30d, 31ab, 33b, 35ab, 36c, 37, 40b, 41ab, 42a, 43b, 44, 47b, 77d, 81ac, 83a, 84a, 94b, 96b, 97a, 99, 101b, 102b, 103b, 104c, 105c, 115a, 116a, 118b, 119b, 120b, 122b, 123c, 133b, 134.

The case weights  $w_i, i = 1, \dots, N$

*< R weights Input 25b >* ≡

```
SEXP weights  
◇
```

Fragment referenced in 30b, 39c, 77c, 80b, 81b, 82, 83b, 84b, 85a, 86, 87b, 89c, 90b, 94a, 95a, 106b, 107a, 110b, 111b, 114b, 115b, 119a, 120a, 123b, 126b.

Defines: **weights** 3b, 4, 5a, 6, 8, 15ab, 17, 19, 25cd, 30d, 31a, 33b, 34b, 35abcd, 36c, 37, 40b, 41a, 49a, 77d, 81ac, 83a, 84ac, 85b, 87a, 88ab, 90a, 91a, 94b, 96b, 97a, 106c, 107b, 111a, 112a, 115a, 116a, 119b, 120b, 123c, 127a.

can be constant one ( $\text{XLENGTH}(\text{weights}) == 0$  or  $\text{weights} = \text{integer}(0)$ ) or integer-valued, with **HAS\_WEIGHTS** == 0 in the former case

*< C integer weights Input 25c >* ≡

```
int *weights,  
int HAS_WEIGHTS,  
◇
```

Fragment referenced in 92ab, 97c, 98a, 100cd, 108cd, 113ab, 117bc, 121cd.

Defines: **HAS\_WEIGHTS** 25d, 93a, 99, 101b, 109b, 114a, 118b, 122b, **weights**, 4, 6, 8, 15b, 19, 25c, 30d, 31a, 33b, 34b, 35abcd, 36c, 37, 40b, 41a, 77d, 81ac, 83a, 84ac, 85b, 87a, 88a, 90a, 94b, 106c, 111a, 115a, 119b, 123c, 127a.

Uses: **weights** 25b.

Case weights larger than **INT\_MAX** are stored as double

*< C real weights Input 25d >* ≡

```
double *weights,  
int HAS_WEIGHTS,  
◇
```

Fragment referenced in 91b, 92c, 97b, 98b, 100b, 101a, 108b, 109a, 112d, 113c, 117a, 118a, 121b, 122a.

Defines: **HAS\_WEIGHTS** 25c, 93a, 99, 101b, 109b, 114a, 118b, 122b, **weights**, 4, 6, 8, 15b, 19, 25c, 30d, 31a, 33b, 34b, 35abcd, 36c, 37, 40b, 41a, 77d, 81ac, 83a, 84ac, 85b, 87a, 88a, 90a, 94b, 106c, 111a, 115a, 119b, 123c, 127a.

Uses: **weights** 25b.

The sum of all case weights is a double

*< C sumweights Input 25e >* ≡

```
double sumweights
◇
```

Fragment referenced in 79, 80a, 81b, 83b.

Defines: `sumweights` 32, 34ab, 35abcd, 43ab, 44, 46, 48, 49bd, 71, 72, 73b, 77a, 79, 80a, 81ac, 83a, 84a, 127a, 145b.

Subsets  $\mathcal{A} \subseteq \{1, \dots, N\}$  are R style indices

*< R subset Input 26a >* ≡

```
SEXP subset
◇
```

Fragment referenced in 30b, 39c, 77c, 80b, 81b, 82, 83b, 84b, 85a, 86, 87b, 89c, 90b, 94a, 95a, 102a, 103a, 106b, 107a, 110b, 111b, 114b, 115b, 119a, 120a, 123b, 124a, 126ab.

Defines: `subset` 3b, 4, 5ab, 6, 8, 15ab, 17, 19, 26de, 30d, 31a, 32, 33b, 34ab, 36c, 37, 40b, 41a, 43b, 44, 77d, 81ac, 83a, 84ac, 85b, 87a, 88ab, 89a, 90a, 91a, 94b, 96b, 97a, 102b, 103b, 104c, 105c, 106c, 107b, 111a, 112a, 115a, 116a, 119b, 120b, 123c, 124b, 126a, 127a, 128ab, 129ab.

are either not existent (`XLENGTH(subset) == 0`) or of length

*< C integer Nsubset Input 26b >* ≡

```
R_xlen_t Nsubset
◇
```

Fragment referenced in 26c, 37, 41a, 81a, 83a, 84c, 87a, 90a, 94b, 102b, 106c, 111a, 115a, 119b, 128ab, 129b.

Defines: `Nsubset` 34b, 37, 41a, 77d, 81ac, 83a, 84ac, 85b, 87a, 88abc, 89a, 90a, 91a, 93a, 94b, 96b, 97a, 102b, 103b, 104c, 105c, 106c, 107b, 111a, 112a, 115a, 116a, 119b, 120b, 128ab, 129b.

Optionally, one can specify a subset of the subset via

*< C subset range Input 26c >* ≡

```
R_xlen_t offset,
< C integer Nsubset Input 26b >
◇
```

Fragment referenced in 26de, 77c, 81b, 83b, 85a, 87b, 90b, 95a, 103a, 107a, 111b, 115b, 120a.

Defines: `offset` 32, 34b, 35abcd, 77d, 81c, 84a, 85b, 88ab, 91a, 96b, 97a, 103b, 104c, 105c, 107b, 112a, 116a, 120b.

where `offset` is a C style index for `subset`.

Subsets are stored either as integer

*< C integer subset Input 26d >* ≡

```
int *subset,
< C subset range Input 26c >
◇
```

Fragment referenced in 92bc, 98ab, 100d, 101a, 104b, 105b, 108d, 109a, 113bc, 117c, 118a, 121d, 122a.

Defines: `subset` 3b, 4, 5ab, 6, 8, 15ab, 17, 19, 26ae, 30d, 31a, 32, 33b, 34ab, 36c, 37, 40b, 41a, 43b, 44, 77d, 81ac, 83a, 84ac, 85b, 87a, 88ab, 89a, 90a, 91a, 94b, 96b, 97a, 102b, 103b, 104c, 105c, 106c, 107b, 111a, 112a, 115a, 116a, 119b, 120b, 123c, 124b, 126a, 127a, 128ab, 129ab.

or double (to allow for indices larger than `INT_MAX`)

$\langle C \text{ real subset Input 26e} \rangle \equiv$

```
double *subset,  
 $\langle C \text{ subset range Input 26c} \rangle$   
◇
```

Fragment referenced in 91b, 92a, 97bc, 100bc, 104a, 105a, 108bc, 112d, 113a, 117ab, 121bc.

Defines: `subset` 3b, 4, 5ab, 6, 8, 15ab, 17, 19, 26ad, 30d, 31a, 32, 33b, 34ab, 36c, 37, 40b, 41a, 43b, 44, 77d, 81ac, 83a, 84ac, 85b, 87a, 88ab, 89a, 90a, 91a, 94b, 96b, 97a, 102b, 103b, 104c, 105c, 106c, 107b, 111a, 112a, 115a, 116a, 119b, 120b, 123c, 124b, 126a, 127a, 128ab, 129ab.

Blocks  $\text{block}_i, i = 1, \dots, N$

$\langle R \text{ block Input 26f} \rangle \equiv$

```
SEXP block  
◇
```

Fragment referenced in 30b, 39c, 47a, 119a, 123b, 124a, 125b, 126a.

Defines: `block` 3b, 4, 5a, 6, 8, 15ab, 17, 19, 27b, 30d, 31ab, 34ab, 36c, 37, 40b, 41ab, 47b, 119b, 120b, 122b, 123c, 124b, 125b, 126a, 145a.

at  $B$  levels

$\langle C \text{ integer } B \text{ Input 27a} \rangle \equiv$

```
int B  
◇
```

Fragment referenced in 27b, 32, 149b, 150.

Defines: `B` 31ab, 32, 33a, 34a, 37, 41ab, 42b, 45, 46, 48, 49b, 70, 71, 74, 119b, 120b, 122b, 132abc, 133ab, 134, 147a, 148, 149b, 150.

are stored as a factor

$\langle C \text{ integer block Input 27b} \rangle \equiv$

```
int *block,  
 $\langle C \text{ integer } B \text{ Input 27a} \rangle$ ,  
◇
```

Fragment referenced in 120c.

Defines: `block` 3b, 4, 5a, 6, 8, 15ab, 17, 19, 26f, 30d, 31ab, 34ab, 36c, 37, 40b, 41ab, 47b, 119b, 120b, 122b, 123c, 124b, 125b, 126a, 145a.

The tabulation of block (potentially in subsets) is

$\langle R \text{ blockTable Input 27c} \rangle \equiv$

```
SEXP blockTable  
◇
```

Fragment referenced in 124a, 125b, 126a.

Defines: `blockTable` 37, 123c, 124b, 125b, 126a.

where the table is of length  $B + 1$  and the first element counts the number of missing values (although these are NOT allowed in block).

### 3.2.1 Example Data and Code

We start with setting-up some toy data sets to be used as test bed. The data over both the 1d and the 2d case, including case weights, subsets and blocks.

```
> N <- 20L
> P <- 3L
> Lx <- 10L
> Ly <- 5L
> Q <- 4L
> B <- 2L
> iX2d <- rbind(0, matrix(runif(Lx * P), nrow = Lx))
> ix <- sample(1:Lx, size = N, replace = TRUE)
> levels(ix) <- 1:Lx
> ixf <- factor(ix, levels = 1:Lx, labels = 1:Lx)
> x <- iX2d[ix + 1,]
> Xfactor <- diag(Lx)[ix,]
> iY2d <- rbind(0, matrix(runif(Ly * Q), nrow = Ly))
> iy <- sample(1:Ly, size = N, replace = TRUE)
> levels(iy) <- 1:Ly
> iyf <- factor(iy, levels = 1:Ly, labels = 1:Ly)
> y <- iY2d[iy + 1,]
> weights <- sample(0:5, size = N, replace = TRUE)
> block <- sample(gl(B, ceiling(N / B))[1:N])
> subset <- sort(sample(1:N, floor(N * 1.5), replace = TRUE))
> subsety <- sample(1:N, floor(N * 1.5), replace = TRUE)
> r1 <- rep(1:ncol(x), ncol(y))
> r1Xfactor <- rep(1:ncol(Xfactor), ncol(y))
> r2 <- rep(1:ncol(y), each = ncol(x))
> r2Xfactor <- rep(1:ncol(y), each = ncol(Xfactor))
```

As a benchmark, we implement linear statistics, their expectation and covariance, taking case weights, subsets and blocks into account, at R level. In a sense, the core of the **libcoin** package is “just” a less memory-hungry and sometimes faster version of this simple function.

```
> LSEC <-
+ function(X, Y, weights = integer(0), subset = integer(0), block = integer(0))
+ {
+   if (length(weights) == 0) weights <- rep.int(1, NROW(X))
+   if (length(subset) == 0) subset <- seq_len(NROW(X))
+
+   X <- X[subset,, drop = FALSE]
+   Y <- Y[subset,, drop = FALSE]
+   weights <- weights[subset]
+
+   if (length(block) == 0) {
+     w. <- sum(weights)
+     wX <- weights * X
+     wY <- weights * Y
+     ExpX <- colSums(wX)
+     ExpY <- colSums(wY) / w.
+     CovX <- crossprod(X, wX)
+     Yc <- t(t(Y) - ExpY)
+     CovY <- crossprod(Yc, weights * Yc) / w.
+     T <- crossprod(X, wY)
+     Exp <- kronecker(ExpY, ExpX)
+     Cov <- w. / (w. - 1) * kronecker(CovY, CovX) -
+       1 / (w. - 1) * kronecker(CovY, tcrossprod(ExpX))
+
+     list(LinearStatistic = as.vector(T), Expectation = as.vector(Exp),
```

```

+           Covariance = Cov, Variance = diag(Cov))
+   } else {
+     block <- block[subset]
+     ret <- list(LinearStatistic = 0, Expectation = 0,
+               Covariance = 0, Variance = 0)
+     for (b in levels(block)) {
+       tmp <- LSEC(X = X, Y = Y, weights = weights, subset = which(block == b))
+       for (l in names(ret)) ret[[l]] <- ret[[l]] + tmp[[l]]
+     }
+     ret
+   }
+ }

> cmpr <-
+ function(ret1, ret2)
+ {
+   if (inherits(ret1, "LinStatExpCov")) {
+     if (!ret1$varonly)
+       ret1$Covariance <- vcov(ret1)
+   }
+   ret1 <- ret1[!sapply(ret1, is.null)]
+   ret2 <- ret2[!sapply(ret2, is.null)]
+   nm1 <- names(ret1)
+   nm2 <- names(ret2)
+   nm <- c(nm1, nm2)
+   nm <- names(table(nm)[table(nm) == 2])
+   isequal(ret1[nm], ret2[nm])
+ }

```

We now compute the linear statistic along with corresponding expectation, variance and covariance for later reuse.

```

> LECVxyws <- LinStatExpCov(x, y, weights = weights, subset = subset)
> LEVxyws <- LinStatExpCov(x, y, weights = weights, subset = subset, varonly = TRUE)

```

The following tests compare the high-level R implementation (function LSEC()) with the 1d and 2d C level implementations in the two situations with and without specification of X (ie, the dummy matrix in the latter case).

```

> ### with X given
> testit <-
+ function(...)
+ {
+   a <- LinStatExpCov(x, y, ...)
+   b <- LSEC(x, y, ...)
+   d <- LinStatExpCov(X = iX2d, ix = ix, Y = iY2d, iy = iy, ...)
+   cmpr(a, b) && cmpr(d, b)
+ }
> stopifnot(
+   testit() && testit(weights = weights) &&
+   testit(subset = subset) && testit(weights = weights, subset = subset) &&
+   testit(block = block) && testit(weights = weights, block = block) &&
+   testit(subset = subset, block = block) &&
+   testit(weights = weights, subset = subset, block = block)
+ )
> ### without dummy matrix X
> testit <-
+ function(...)
+ {
+   a <- LinStatExpCov(X = ix, y, ...)

```

```

+   b <- LSEC(Xfactor, y, ...)
+   d <- LinStatExpCov(X = integer(0), ix = ix, Y = iY2d, iy = iy, ...)
+   cmpr(a, b) && cmpr(d, b)
+ }
> stopifnot(
+   testit() && testit(weights = weights) &&
+   testit(subset = subset) && testit(weights = weights, subset = subset) &&
+   testit(block = block) && testit(weights = weights, block = block) &&
+   testit(subset = subset, block = block) &&
+   testit(weights = weights, subset = subset, block = block)
+ )

```

All three implementations give the same results.

### 3.3 Conventions

Functions starting with `R_` are C functions callable via `.Call()` from R. That means they all return SEXP. These functions allocate memory handled by R.

Functions starting with `RC_` are C functions with SEXP or pointer arguments and possibly an SEXP return value.

Functions starting with `C_` are C functions with pointer arguments only and return a scalar or nothing.

Return values (arguments modified by a function) are named `ans`, sometimes with dimension (for example: `PQ_ans`).

### 3.4 C User Interface

#### 3.4.1 One-Dimensional Case (“1d”)

*< User Interface 30a >* ≡

```

< RC_ExpectationCovarianceStatistic 32 >
< R_ExpectationCovarianceStatistic 31a >
< R_PermutedLinearStatistic 37 >
< R_StandardisePermutedLinearStatistic 39a >
◇

```

Fragment referenced in [23a](#).

The data are given as  $\mathbf{x}_i$  and  $\mathbf{y}_i$  for  $i = 1, \dots, N$ , optionally with case weights, subset and blocks. The latter three variables are ignored when specified as `integer(0)`.

*< User Interface Input 30b >* ≡

```

< R x Input 23d >
< R y Input 24d >
< R weights Input 25b >,
< R subset Input 26a >,
< R block Input 26f >,
◇

```

Fragment referenced in [30c](#), [32](#), [36b](#).

*< R\_ExpectationCovarianceStatistic Prototype 30c >* ≡

```
SEXP R_ExpectationCovarianceStatistic
(
  < User Interface Input 30b >
  SEXP varonly,
  SEXP tol
)
◇
```

Fragment referenced in [22b](#), [31a](#).

Uses: [R\\_ExpectationCovarianceStatistic 31a](#).

This function can be called from other packages.

"libcoinAPI.h" [30d](#)≡

```
< C Header 154b >
#include <R_ext/Rdynload.h>
#include <libcoin.h>

extern SEXP libcoin_R_ExpectationCovarianceStatistic(
  SEXP x, SEXP y, SEXP weights, SEXP subset, SEXP block, SEXP varonly,
  SEXP tol
) {
  static SEXP(*fun)(SEXP, SEXP, SEXP, SEXP, SEXP, SEXP, SEXP) = NULL;
  if (fun == NULL)
    fun = (SEXP*)(SEXP, SEXP, SEXP, SEXP, SEXP, SEXP, SEXP)
      R_GetCCallable("libcoin", "R_ExpectationCovarianceStatistic");
  return fun(x, y, weights, subset, block, varonly, tol);
}
◇
```

File defined by [30d](#), [36c](#), [38c](#), [40b](#), [47b](#), [50c](#), [53](#), [55b](#), [61a](#), [132c](#), [136a](#), [138b](#), [140b](#).

Uses: [block 26f](#), [27b](#), [R\\_ExpectationCovarianceStatistic 31a](#), [subset 26ade](#), [weights 25b](#), [weights, 25cd](#), [x 23d](#), [24bc](#), [y 24df](#), [25a](#).

The C interface essentially sets-up the necessary memory and calls a C level function for the computations.

*< R\_ExpectationCovarianceStatistic 31a >* ≡

```
< R_ExpectationCovarianceStatistic Prototype 30c >
{
  SEXP ans;

  < Setup Dimensions 31b >

  PROTECT(ans = RC_init_LECV_1d(P, Q, INTEGER(varonly)[0], B, TYPEOF(x) == INTSXP, REAL(tol)[0]));

  RC_ExpectationCovarianceStatistic(x, y, weights, subset, block, ans);

  UNPROTECT(1);
  return(ans);
}
◇
```

Fragment referenced in [30a](#).

Defines: [R\\_ExpectationCovarianceStatistic 6](#), [30cd](#), [152c](#), [153](#).

Uses: [B 27a](#), [block 26f](#), [27b](#), [P 24a](#), [Q 24e](#), [RC\\_ExpectationCovarianceStatistic 32](#), [45](#), [RC\\_init\\_LECV\\_1d 149b](#), [subset 26ade](#), [weights 25b](#), [weights, 25cd](#), [x 23d](#), [24bc](#), [y 24df](#), [25a](#).

$P$ ,  $Q$  and  $B$  are first extracted from the data. The case where  $X$  is an implicitly specified dummy matrix, the dimension  $P$  is the number of levels of  $x$ .

*Setup Dimensions 31b* ≡

```
int P, Q, B;

if (typeof(x) == INTSXP) {
    P = NLEVELS(x);
} else {
    P = NCOL(x);
}
Q = NCOL(y);

B = 1;
if (LENGTH(block) > 0)
    B = NLEVELS(block);
◇
```

Fragment referenced in [31a](#), [37](#).

Uses: B [27a](#), block [26f](#), [27b](#), NCOL [130c](#), NLEVELS [131a](#), P [24a](#), Q [24e](#), x [23d](#), [24bc](#), y [24df](#), [25a](#).

The core function first computes the linear statistic (as there is no need to pay attention to blocks) and, in a second step, starts a loop over potential blocks.

FIXME: `x` being an integer (`Xfactor`) with some 0 elements is not handled correctly (as `sumweights` doesn't take this information into account; use `subset` to exclude these missings (as done in `LinStatExpCov()`)

*< RC\_ExpectationCovarianceStatistic 32 >* ≡

```
void RC_ExpectationCovarianceStatistic
(
  < User Interface Input 30b >
  SEXP ans
) {
  < C integer N Input 23c >;
  < C integer P Input 24a >;
  < C integer Q Input 24e >;
  < C integer B Input 27a >;
  double *sumweights, *table;
  double *ExpInf, *VarInf, *CovInf, *ExpX, *ExpXtotal, *VarX, *CovX;
  double *tmpV, *tmpCV;
  SEXP nullvec, subset_block;

  < Extract Dimensions 33a >

  < Compute Linear Statistic 33b >

  < Setup Memory and Subsets in Blocks 34a >

  /* start with subset[0] */
  R_xlen_t offset = (R_xlen_t) table[0];

  for (int b = 0; b < B; b++) {

    < Compute Sum of Weights in Block 34b >

    /* don't do anything for empty blocks or blocks with weight 1 */
    if (sumweights[b] > 1) {

      < Compute Expectation Linear Statistic 35a >

      < Compute Covariance Influence 35b >

      if (C_get_varonly(ans)) {
        < Compute Variance Linear Statistic 35c >
      } else {
        < Compute Covariance Linear Statistic 35d >
      }
    }

    /* next iteration starts with subset[cumsum(table[1:(b + 1)])] */
    offset += (R_xlen_t) table[b + 1];
  }

  < Compute Variance from Covariance 36a >

  R_Free(ExpX); R_Free(VarX); R_Free(CovX);
  UNPROTECT(2);
}
◇
```

Fragment referenced in [30a](#).

Defines: [RC\\_ExpectationCovarianceStatistic 31a](#).

Uses: [B 27a](#), [C\\_get\\_varonly 142b](#), [offset 26c](#), [subset 26ade](#), [sumweights 25e](#).

The dimensions are available from the return object:

*⟨ Extract Dimensions 33a ⟩* ≡

```
P = C_get_P(ans);
Q = C_get_Q(ans);
N = NROW(x);
B = C_get_B(ans);
◇
```

Fragment referenced in 32.

Uses: B 27a, C\_get\_B 146a, C\_get\_P 141c, C\_get\_Q 142a, N 23bc, NROW 130b, P 24a, Q 24e, x 23d, 24bc.

The linear statistic  $\mathbf{T}(\mathcal{A})$  can be computed without taking blocks into account.

*⟨ Compute Linear Statistic 33b ⟩* ≡

```
RC_LinearStatistic(x, N, P, REAL(y), Q, weights, subset,
                  Offset0, XLENGTH(subset),
                  C_get_LinearStatistic(ans));
◇
```

Fragment referenced in 32.

Uses: C\_get\_LinearStatistic 142d, N 23bc, Offset0 21b, P 24a, Q 24e, RC\_LinearStatistic 77d, subset 26ade, weights 25b, weights, 25cd, x 23d, 24bc, y 24df, 25a.

We next extract memory from the return object and allocate some additional memory. The most important step is to tabulate blocks and to order the subset with respect to blocks. In absence of block, this just returns subset.

⟨ Setup Memory and Subsets in Blocks 34a ⟩ ≡

```

ExpInf = C_get_ExpectationInfluence(ans);
VarInf = C_get_VarianceInfluence(ans);
CovInf = C_get_CovarianceInfluence(ans);
ExpXtotal = C_get_ExpectationX(ans);
for (int p = 0; p < P; p++) ExpXtotal[p] = 0.0;
ExpX = R_Calloc(P, double);
/* Fix by Joanidis Kristoforos: P > INT_MAX is possible
   for maximally selected statistics (when X is an integer).
   2018-12-13 */
if (C_get_varonly(ans)) {
    VarX = R_Calloc(P, double);
    CovX = R_Calloc(1, double);
} else {
    VarX = R_Calloc(1, double);
    CovX = R_Calloc(PP12(P), double);
}
table = C_get_TableBlock(ans);
sumweights = C_get_Sumweights(ans);
PROTECT(nullvec = allocVector(INTSXP, 0));

if (B == 1) {
    table[0] = 0.0;
    table[1] = RC_Sums(N, nullvec, subset, Offset0, XLENGTH(subset));
} else {
    RC_OneTableSums(INTEGER(block), N, B + 1, nullvec, subset, Offset0,
                    XLENGTH(subset), table);
}
if (table[0] > 0)
    error("No missing values allowed in block");
PROTECT(subset_block = RC_order_subset_wrt_block(N, subset, block,
                                                VECTOR_ELT(ans, TableBlock_SLOT)));
◇

```

Fragment referenced in 32.

Uses: B 27a, block 26f, 27b, C\_get\_CovarianceInfluence 144c, C\_get\_ExpectationInfluence 144b, C\_get\_ExpectationX 144a, C\_get\_Sumweights 145b, C\_get\_TableBlock 145a, C\_get\_VarianceInfluence 144d, C\_get\_varonly 142b, N 23bc, Offset0 21b, P 24a, PP12 131b, RC\_OneTableSums 112a, RC\_order\_subset\_wrt\_block 124b, RC\_Sums 91a, subset 26ade, sumweights 25e, TableBlock\_SLOT 21b.

We compute  $\mu(\mathcal{A})$  based on  $\mathbb{E}(h \mid S(\mathcal{A}))$  and  $\sum_{i \in \mathcal{A}} w_i \mathbf{x}_i$  for the subset given by subset and the  $b$ th level of block. The expectation is initialised zero when  $b = 0$  and values add-up over blocks.

⟨ Compute Sum of Weights in Block 34b ⟩ ≡

```

/* compute sum of case weights in block b of subset */
if (table[b + 1] > 0) {
    sumweights[b] = RC_Sums(N, weights, subset_block,
                           offset, (R_xlen_t) table[b + 1]);
} else {
    /* offset = something and Nsubset = 0 means Nsubset = N in
       RC_Sums; catch empty or zero-weight block levels here */
    sumweights[b] = 0.0;
}
◇

```

Fragment referenced in 32.

Uses: block 26f, 27b, N 23bc, Nsubset 26b, offset 26c, RC\_Sums 91a, subset 26ade, sumweights 25e, weights 25b, weights, 25cd.

*< Compute Expectation Linear Statistic 35a > ≡*

```
RC_ExpectationInfluence(N, y, Q, weights, subset_block, offset,
                        (R_xlen_t) table[b + 1], sumweights[b], ExpInf + b * Q);
RC_ExpectationX(x, N, P, weights, subset_block, offset,
               (R_xlen_t) table[b + 1], ExpX);
for (int p = 0; p < P; p++) ExpXtotal[p] += ExpX[p];
C_ExpectationLinearStatistic(P, Q, ExpInf + b * Q, ExpX, b,
                             C_get_Expectation(ans));
◇
```

Fragment referenced in 32.

Uses: C\_ExpectationLinearStatistic 78b, C\_get\_Expectation 143a, N 23bc, offset 26c, P 24a, Q 24e,  
RC\_ExpectationInfluence 81c, RC\_ExpectationX 85b, sumweights 25e, weights 25b, weights, 25cd, x 23d, 24bc,  
y 24df, 25a.

The covariance  $\mathbb{V}(h \mid S(\mathcal{A}))$  is now computed for the subset given by subset and the  $b$ th level of block. Note that CovInf stores the values for each block in the return object (for later reuse).

*< Compute Covariance Influence 35b > ≡*

```
/* C_ordered_Xfactor and C_unordered_Xfactor need both VarInf and CovInf */
RC_CovarianceInfluence(N, y, Q, weights, subset_block, offset,
                       (R_xlen_t) table[b + 1], ExpInf + b * Q, sumweights[b],
                       !DoVarOnly, CovInf + b * Q * (Q + 1) / 2);
/* extract variance from covariance */
tmpCV = CovInf + b * Q * (Q + 1) / 2;
tmpV = VarInf + b * Q;
for (int q = 0; q < Q; q++) tmpV[q] = tmpCV[S(q, q, Q)];
◇
```

Fragment referenced in 32.

Uses: C\_ordered\_Xfactor 70, C\_unordered\_Xfactor 74, DoVarOnly 21b, N 23bc, offset 26c, Q 24e,  
RC\_CovarianceInfluence 84a, S 21a, sumweights 25e, weights 25b, weights, 25cd, y 24df, 25a.

We can now compute the variance or covariance of the linear statistic  $\Sigma(\mathcal{A})$ :

*< Compute Variance Linear Statistic 35c > ≡*

```
RC_CovarianceX(x, N, P, weights, subset_block, offset,
               (R_xlen_t) table[b + 1], ExpX, DoVarOnly, VarX);
C_VarianceLinearStatistic(P, Q, VarInf + b * Q, ExpX, VarX, sumweights[b],
                           b, C_get_Variance(ans));
◇
```

Fragment referenced in 32.

Uses: C\_get\_Variance 143b, C\_VarianceLinearStatistic 80a, DoVarOnly 21b, N 23bc, offset 26c, P 24a, Q 24e,  
RC\_CovarianceX 88a, sumweights 25e, weights 25b, weights, 25cd, x 23d, 24bc.

*< Compute Covariance Linear Statistic 35d > ≡*

```
RC_CovarianceX(x, N, P, weights, subset_block, offset,
               (R_xlen_t) table[b + 1], ExpX, !DoVarOnly, CovX);
C_CovarianceLinearStatistic(P, Q, CovInf + b * Q * (Q + 1) / 2,
                             ExpX, CovX, sumweights[b], b,
                             C_get_Covariance(ans));
◇
```

Fragment referenced in 32.

Uses: C\_CovarianceLinearStatistic 79, C\_get\_Covariance 143c, DoVarOnly 21b, N 23bc, offset 26c, P 24a, Q 24e,  
RC\_CovarianceX 88a, sumweights 25e, weights 25b, weights, 25cd, x 23d, 24bc.

*⟨ Compute Variance from Covariance 36a ⟩* ≡

```
/* always return variances */
if (!C_get_varonly(ans)) {
    for (int p = 0; p < mPQB(P, Q, 1); p++)
        C_get_Variance(ans)[p] = C_get_Covariance(ans)[S(p, p, mPQB(P, Q, 1))];
}
◇
```

Fragment referenced in [32](#).

Uses: [C\\_get\\_Covariance 143c](#), [C\\_get\\_Variance 143b](#), [C\\_get\\_varonly 142b](#), [mPQB 132a](#), [P 24a](#), [Q 24e](#), [S 21a](#).

The computation of permuted linear statistics is done outside this general function. The user interface is the same, except for an additional number of permutations to be specified.

*⟨ R\_PermutedLinearStatistic Prototype 36b ⟩* ≡

```
SEXP R_PermutedLinearStatistic
(
    ⟨ User Interface Input 30b ⟩
    SEXP nresample
)
◇
```

Fragment referenced in [22b](#), [37](#).

Uses: [R\\_PermutedLinearStatistic 37](#).

This function can be called from other packages.

"libcoinAPI.h" [36c](#) ≡

```
extern SEXP libcoin_R_PermutedLinearStatistic(
    SEXP x, SEXP y, SEXP weights, SEXP subset, SEXP block, SEXP nresample
) {
    static SEXP(*fun)(SEXP, SEXP, SEXP, SEXP, SEXP, SEXP) = NULL;
    if (fun == NULL)
        fun = (SEXP*)(SEXP, SEXP, SEXP, SEXP, SEXP, SEXP)
            R_GetCCallable("libcoin", "R_PermutedLinearStatistic");
    return fun(x, y, weights, subset, block, nresample);
}
◇
```

File defined by [30d](#), [36c](#), [38c](#), [40b](#), [47b](#), [50c](#), [53](#), [55b](#), [61a](#), [132c](#), [136a](#), [138b](#), [140b](#).

Uses: [block 26f](#), [27b](#), [R\\_PermutedLinearStatistic 37](#), [subset 26ade](#), [weights 25b](#), [weights, 25cd](#), [x 23d](#), [24bc](#), [y 24df](#), [25a](#).

The dimensions are extracted from the data in the same ways as above. The function differentiates between the absence and presence of blocks. Case weights are removed by expanding subset accordingly. Once within-block permutations were set-up the Kronecker product of **X** and **Y** is computed. Note that this function returns the matrix of permuted linear statistics; the R interface assigns this matrix to the corresponding element of the `LinStatExpCov` object (because we are not allowed to modify existing R objects at C level).

$\langle R\_PermutedLinearStatistic\ 37 \rangle \equiv$

```
 $\langle R\_PermutedLinearStatistic\ Prototype\ 36b \rangle$ 
{
  SEXP ans, expand_subset, block_subset, perm, tmp, blockTable;
  double *linstat;
  int PQ;
   $\langle C\ integer\ N\ Input\ 23c \rangle$ ;
   $\langle C\ integer\ Nsubset\ Input\ 26b \rangle$ ;
  R_xlen_t inresample;

   $\langle Setup\ Dimensions\ 31b \rangle$ 
  PQ = mPQB(P, Q, 1);
  N = NROW(y);
  inresample = (R_xlen_t) REAL(nresample)[0];

  PROTECT(ans = allocMatrix(REALSXP, PQ, inresample));
  PROTECT(expand_subset = RC_setup_subset(N, weights, subset));
  Nsubset = XLENGTH(expand_subset);
  PROTECT(tmp = allocVector(REALSXP, Nsubset));
  PROTECT(perm = allocVector(REALSXP, Nsubset));

  GetRNGstate();
  if (B == 1) {
    for (R_xlen_t np = 0; np < inresample; np++) {
       $\langle Setup\ Linear\ Statistic\ 38a \rangle$ 
      C_doPermute(REAL(expand_subset), Nsubset, REAL(tmp), REAL(perm));
      RC_KronSums_Permutation(x, NROW(x), P, REAL(y), Q, expand_subset,
                             Offset0, Nsubset, perm, linstat);
    }
  } else {
    PROTECT(blockTable = allocVector(REALSXP, B + 1));
    /* same as RC_OneTableSums(block, noweights, expand_subset) */
    RC_OneTableSums(INTEGER(block), XLENGTH(block), B + 1, weights, subset, Offset0,
                    XLENGTH(subset), REAL(blockTable));
    PROTECT(block_subset = RC_order_subset_wrt_block(XLENGTH(block), expand_subset,
                                                      block, blockTable));

    for (R_xlen_t np = 0; np < inresample; np++) {
       $\langle Setup\ Linear\ Statistic\ 38a \rangle$ 
      C_doPermuteBlock(REAL(block_subset), Nsubset, REAL(blockTable),
                       B + 1, REAL(tmp), REAL(perm));
      RC_KronSums_Permutation(x, NROW(x), P, REAL(y), Q, block_subset,
                              Offset0, Nsubset, perm, linstat);
    }
    UNPROTECT(2);
  }
  PutRNGstate();

  UNPROTECT(4);
  return(ans);
}
◇
```

Fragment referenced in 30a.

Defines: R\_PermutedLinearStatistic 6, 36bc, 152c, 153.

Uses: B 27a, block 26f, 27b, blockTable 27c, C\_doPermute 128b, C\_doPermuteBlock 129b, mPQB 132a, N 23bc, NROW 130b, Nsubset 26b, Offset0 21b, P 24a, Q 24e, RC\_KronSums\_Permutation 103b, RC\_OneTableSums 112a, RC\_order\_subset\_wrt\_block 124b, RC\_setup\_subset 127a, subset 26ade, weights 25b, weights, 25cd, x 23d, 24bc, y 24df, 25a.

*< Setup Linear Statistic 38a >* ≡

```
if (np % 256 == 0) R_CheckUserInterrupt();
linstat = REAL(ans) + PQ * np;
for (int p = 0; p < PQ; p++)
    linstat[p] = 0.0;
◇
```

Fragment referenced in [37](#), [48](#).

This small function takes an object containing permuted linear statistics and returns the matrix of standardised linear statistics.

*< R\_StandardisePermutedLinearStatistic Prototype 38b >* ≡

```
SEXP R_StandardisePermutedLinearStatistic
(
    SEXP LECV
)
◇
```

Fragment referenced in [22b](#), [39a](#).

Uses: [LECV 141b](#).

This function can be called from other packages.

"libcoinAPI.h" 38c ≡

```
extern SEXP libcoin_R_StandardisePermutedLinearStatistic(
    SEXP LECV
) {
    static SEXP(*fun)(SEXP) = NULL;
    if (fun == NULL)
        fun = (SEXP*)(SEXP)
            R_GetCCallable("libcoin", "R_StandardisePermutedLinearStatistic");
    return fun(LECV);
}
◇
```

File defined by [30d](#), [36c](#), [38c](#), [40b](#), [47b](#), [50c](#), [53](#), [55b](#), [61a](#), [132c](#), [136a](#), [138b](#), [140b](#).

Uses: [LECV 141b](#).

*< R\_StandardisePermutedLinearStatistic 39a >* ≡

```
< R_StandardisePermutedLinearStatistic Prototype 38b >
{
  SEXP ans;
  R_xlen_t nresample = C_get_nresample(LECV);
  double *ls;
  if (!nresample) return(R_NilValue);
  int PQ = C_get_P(LECV) * C_get_Q(LECV);

  PROTECT(ans = allocMatrix(REALSXP, PQ, nresample));

  for (R_xlen_t np = 0; np < nresample; np++) {
    ls = REAL(ans) + PQ * np;
    /* copy first; standarisation is in place */
    for (int p = 0; p < PQ; p++)
      ls[p] = C_get_PermutedLinearStatistic(LECV)[p + PQ * np];
    if (C_get_varonly(LECV)) {
      C_standardise(PQ, ls, C_get_Expectation(LECV),
                   C_get_Variance(LECV), 1, C_get_tol(LECV));
    } else {
      C_standardise(PQ, ls, C_get_Expectation(LECV),
                   C_get_Covariance(LECV), 0, C_get_tol(LECV));
    }
  }
  UNPROTECT(1);
  return(ans);
}
◇
```

Fragment referenced in [30a](#).

Uses: [C\\_get\\_Covariance 143c](#), [C\\_get\\_Expectation 143a](#), [C\\_get\\_nresample 146b](#), [C\\_get\\_P 141c](#),  
[C\\_get\\_PermutedLinearStatistic 146c](#), [C\\_get\\_Q 142a](#), [C\\_get\\_tol 146d](#), [C\\_get\\_Variance 143b](#), [C\\_get\\_varonly 142b](#),  
[C\\_standardise 64a](#), [LECV 141b](#).

### 3.4.2 Two-Dimensional Case (“2d”)

*< 2d User Interface 39b >* ≡

```
< RC_ExpectationCovarianceStatistic_2d 45 >
< R_ExpectationCovarianceStatistic_2d 41a >
< R_PermutedLinearStatistic_2d 48 >
◇
```

Fragment referenced in [23a](#).

*< 2d User Interface Input 39c >* ≡

```
< R x Input 23d >
SEXP ix,
< R y Input 24d >
SEXP iy,
< R weights Input 25b >,
< R subset Input 26a >,
< R block Input 26f >,
◇
```

Fragment referenced in [40a](#), [45](#).

*< R\_ExpectationCovarianceStatistic\_2d Prototype 40a >* ≡

```
SEXP R_ExpectationCovarianceStatistic_2d
(
  < 2d User Interface Input 39c >
  SEXP varonly,
  SEXP tol
)
◇
```

Fragment referenced in [22b](#), [41a](#).

Uses: [R\\_ExpectationCovarianceStatistic\\_2d 41a](#).

This function can be called from other packages.

"libcoinAPI.h" [40b](#)≡

```
extern SEXP libcoin_R_ExpectationCovarianceStatistic_2d(
  SEXP x, SEXP ix, SEXP y, SEXP iy, SEXP weights, SEXP subset, SEXP block,
  SEXP varonly, SEXP tol
) {
  static SEXP(*fun)(SEXP, SEXP, SEXP, SEXP, SEXP, SEXP, SEXP, SEXP, SEXP) = NULL;
  if (fun == NULL)
    fun = (SEXP*)(SEXP, SEXP, SEXP, SEXP, SEXP, SEXP, SEXP, SEXP, SEXP)
      R_GetCCallable("libcoin", "R_ExpectationCovarianceStatistic_2d");
  return fun(x, ix, y, iy, weights, subset, block, varonly, tol);
}
◇
```

File defined by [30d](#), [36c](#), [38c](#), [40b](#), [47b](#), [50c](#), [53](#), [55b](#), [61a](#), [132c](#), [136a](#), [138b](#), [140b](#).

Uses: [block 26f](#), [27b](#), [R\\_ExpectationCovarianceStatistic\\_2d 41a](#), [subset 26ade](#), [weights 25b](#), [weights, 25cd](#), [x 23d](#), [24bc](#), [y 24df](#), [25a](#).

$\langle R\_ExpectationCovarianceStatistic\_2d\ 41a \rangle \equiv$

$\langle R\_ExpectationCovarianceStatistic\_2d\ Prototype\ 40a \rangle$

```
{
  SEXP ans;
   $\langle C\ integer\ N\ Input\ 23c \rangle$ ;
   $\langle C\ integer\ Nsubset\ Input\ 26b \rangle$ ;
  int Xfactor;

  N = XLENGTH(ix);
  Nsubset = XLENGTH(subset);
  Xfactor = XLENGTH(x) == 0;

   $\langle Setup\ Dimensions\ 2d\ 41b \rangle$ 

  PROTECT(ans = RC_init_LECV_2d(P, Q, INTEGER(varonly)[0],
                                Lx, Ly, B, Xfactor, REAL(tol)[0]));

  if (B == 1) {
    RC_TwoTableSums(INTEGER(ix), N, Lx + 1, INTEGER(iy), Ly + 1,
                    weights, subset, Offset0, Nsubset,
                    C_get_Table(ans));
  } else {
    RC_ThreeTableSums(INTEGER(ix), N, Lx + 1, INTEGER(iy), Ly + 1,
                      INTEGER(block), B, weights, subset, Offset0, Nsubset,
                      C_get_Table(ans));
  }
  RC_ExpectationCovarianceStatistic_2d(x, ix, y, iy, weights,
                                       subset, block, ans);

  UNPROTECT(1);
  return(ans);
}
◇
```

Fragment referenced in [39b](#).

Defines: [R\\_ExpectationCovarianceStatistic\\_2d 8, 40ab, 152c, 153](#).

Uses: [B 27a, block 26f, 27b, C\\_get\\_Table 145c, N 23bc, Nsubset 26b, Offset0 21b, P 24a, Q 24e, RC\\_init\\_LECV\\_2d 150, RC\\_ThreeTableSums 120b, RC\\_TwoTableSums 116a, subset 26ade, weights 25b, weights, 25cd, x 23d, 24bc, y 24df, 25a](#).

$\langle Setup\ Dimensions\ 2d\ 41b \rangle \equiv$

```
int P, Q, B, Lx, Ly;

if (XLENGTH(x) == 0) {
  P = NLEVELS(ix);
} else {
  P = NCOL(x);
}
Q = NCOL(y);

B = 1;
if (XLENGTH(block) > 0)
  B = NLEVELS(block);

Lx = NLEVELS(ix);
Ly = NLEVELS(iy);
◇
```

Fragment referenced in [41a, 48](#).

Uses: [B 27a, block 26f, 27b, NCOL 130c, NLEVELS 131a, P 24a, Q 24e, x 23d, 24bc, y 24df, 25a](#).

*< Linear Statistic 2d 42a >* ≡

```
if (Xfactor) {
  for (int j = 1; j < Lyp1; j++) { /* j = 0 means NA */
    for (int i = 1; i < Lxp1; i++) { /* i = 0 means NA */
      for (int q = 0; q < Q; q++)
        linstat[q * (Lxp1 - 1) + (i - 1)] +=
          btab[j * Lxp1 + i] * REAL(y)[q * Lyp1 + j];
    }
  }
} else {
  for (int p = 0; p < P; p++) {
    for (int q = 0; q < Q; q++) {
      int qPp = q * P + p;
      int qLy = q * Lyp1;
      for (int i = 0; i < Lxp1; i++) {
        int pLxi = p * Lxp1 + i;
        for (int j = 0; j < Lyp1; j++)
          linstat[qPp] += REAL(y)[qLy + j] * REAL(x)[pLxi] * btab[j * Lxp1 + i];
      }
    }
  }
}
◇
```

Fragment referenced in [45](#), [49d](#).

Uses: [P 24a](#), [Q 24e](#), [x 23d](#), [24bc](#), [y 24df](#), [25a](#).

*< 2d Total Table 42b >* ≡

```
for (int i = 0; i < Lxp1 * Lyp1; i++)
  table2d[i] = 0.0;
for (int b = 0; b < B; b++) {
  for (int i = 0; i < Lxp1; i++) {
    for (int j = 0; j < Lyp1; j++)
      table2d[j * Lxp1 + i] += table[b * Lxp1 * Lyp1 + j * Lxp1 + i];
  }
}
◇
```

Fragment referenced in [45](#).

Uses: [B 27a](#).

⟨ *Col Row Total Sums* 43a ⟩ ≡

```
/* Remember: first row / column count NAs */
/* column sums */
for (int q = 1; q < Lxp1; q++) {
  csum[q] = 0;
  for (int p = 1; p < Lxp1; p++)
    csum[q] += btab[q * Lxp1 + p];
}
csum[0] = 0; /* NA */
/* row sums */
for (int p = 1; p < Lxp1; p++) {
  rsum[p] = 0;
  for (int q = 1; q < Lxp1; q++)
    rsum[p] += btab[q * Lxp1 + p];
}
rsum[0] = 0; /* NA */
/* total sum */
sumweights[b] = 0;
for (int i = 1; i < Lxp1; i++) sumweights[b] += rsum[i];
◇
```

Fragment referenced in [45](#), [48](#).

Uses: [sumweights](#) [25e](#).

⟨ *2d Expectation* 43b ⟩ ≡

```
RC_ExpectationInfluence(NROW(y), y, Q, Rcsum, subset, Offset0, 0, sumweights[b], ExpInf);

if (LENGTH(x) == 0) {
  for (int p = 0; p < P; p++)
    ExpX[p] = rsum[p + 1];
} else {
  RC_ExpectationX(x, NROW(x), P, Rrsum, subset, Offset0, 0, ExpX);
}

C_ExpectationLinearStatistic(P, Q, ExpInf, ExpX, b, C_get_Expectation(ans));
◇
```

Fragment referenced in [45](#).

Uses: [C\\_ExpectationLinearStatistic](#) [78b](#), [C\\_get\\_Expectation](#) [143a](#), [NROW](#) [130b](#), [Offset0](#) [21b](#), [P](#) [24a](#), [Q](#) [24e](#),  
[RC\\_ExpectationInfluence](#) [81c](#), [RC\\_ExpectationX](#) [85b](#), [subset](#) [26ade](#), [sumweights](#) [25e](#), [x](#) [23d](#), [24bc](#), [y](#) [24df](#), [25a](#).

*< 2d Covariance 44 >* ≡

```
/* C_ordered_Xfactor needs both VarInf and CovInf */
RC_CovarianceInfluence(NROW(y), y, Q, Rcsum, subset, Offset0, 0, ExpInf, sumweights[b],
    !DoVarOnly, C_get_CovarianceInfluence(ans));
for (int q = 0; q < Q; q++)
    C_get_VarianceInfluence(ans)[q] = C_get_CovarianceInfluence(ans)[S(q, q, Q)];

if (C_get_varonly(ans)) {
    if (LENGTH(x) == 0) {
        for (int p = 0; p < P; p++) CovX[p] = ExpX[p];
    } else {
        RC_CovarianceX(x, NROW(x), P, Rrsum, subset, Offset0, 0, ExpX, DoVarOnly, CovX);
    }
    C_VarianceLinearStatistic(P, Q, C_get_VarianceInfluence(ans),
        ExpX, CovX, sumweights[b], b,
        C_get_Variance(ans));
} else {
    if (LENGTH(x) == 0) {
        for (int p = 0; p < PP12(P); p++) CovX[p] = 0.0;
        for (int p = 0; p < P; p++) CovX[S(p, p, P)] = ExpX[p];
    } else {
        RC_CovarianceX(x, NROW(x), P, Rrsum, subset, Offset0, 0, ExpX, !DoVarOnly, CovX);
    }
    C_CovarianceLinearStatistic(P, Q, C_get_CovarianceInfluence(ans),
        ExpX, CovX, sumweights[b], b,
        C_get_Covariance(ans));
}
◇
```

Fragment referenced in 45.

Uses: C\_CovarianceLinearStatistic 79, C\_get\_Covariance 143c, C\_get\_CovarianceInfluence 144c,  
C\_get\_Variance 143b, C\_get\_VarianceInfluence 144d, C\_get\_varonly 142b, C\_ordered\_Xfactor 70,  
C\_VarianceLinearStatistic 80a, DoVarOnly 21b, NROW 130b, Offset0 21b, P 24a, PP12 131b, Q 24e,  
RC\_CovarianceInfluence 84a, RC\_CovarianceX 88a, S 21a, subset 26ade, sumweights 25e, x 23d, 24bc, y 24df, 25a.

⟨ *RC\_ExpectationCovarianceStatistic\_2d 45* ⟩ ≡

```
void RC_ExpectationCovarianceStatistic_2d
(
  ⟨ 2d User Interface Input 39c ⟩
  SEXP ans
) {
  ⟨ 2d Memory 46 ⟩

  ⟨ 2d Total Table 42b ⟩

  linstat = C_get_LinearStatistic(ans);
  for (int p = 0; p < mPQB(P, Q, 1); p++)
    linstat[p] = 0.0;

  for (int b = 0; b < B; b++) {
    btab = table + Lxp1 * Lyp1 * b;

    ⟨ Linear Statistic 2d 42a ⟩

    ⟨ Col Row Total Sums 43a ⟩

    ⟨ 2d Expectation 43b ⟩

    ⟨ 2d Covariance 44 ⟩
  }

  /* always return variances */
  if (!C_get_varonly(ans)) {
    for (int p = 0; p < mPQB(P, Q, 1); p++)
      C_get_Variance(ans)[p] = C_get_Covariance(ans)[S(p, p, mPQB(P, Q, 1))];
  }

  R_Free(CovX);
  R_Free(table2d);
  UNPROTECT(2);
}
◇
```

Fragment referenced in [39b](#).

Defines: [RC\\_ExpectationCovarianceStatistic 31a, 32](#).

Uses: [B 27a](#), [C\\_get\\_Covariance 143c](#), [C\\_get\\_LinearStatistic 142d](#), [C\\_get\\_Variance 143b](#), [C\\_get\\_varonly 142b](#),  
[mPQB 132a](#), [P 24a](#), [Q 24e](#), [S 21a](#).

( 2d Memory 46 ) ≡

```
SEXP Rcsum, Rrsum;
int P, Q, Lxp1, Lyp1, B, Xfactor;
double *ExpInf, *ExpX, *CovX;
double *table, *table2d, *csum, *rsum, *sumweights, *btab, *linstat;

P = C_get_P(ans);
Q = C_get_Q(ans);

ExpInf = C_get_ExpectationInfluence(ans);
ExpX = C_get_ExpectationX(ans);
table = C_get_Table(ans);
sumweights = C_get_Sumweights(ans);

Lxp1 = C_get_dimTable(ans)[0];
Lyp1 = C_get_dimTable(ans)[1];
B = C_get_B(ans);
Xfactor = C_get_Xfactor(ans);

if (C_get_varonly(ans)) {
  CovX = R_Calloc(P, double);
} else {
  CovX = R_Calloc(PP12(P), double);
}

table2d = R_Calloc(Lxp1 * Lyp1, double);
PROTECT(Rcsum = allocVector(REALSXP, Lyp1));
csum = REAL(Rcsum);
PROTECT(Rrsum = allocVector(REALSXP, Lxp1));
rsum = REAL(Rrsum);
◇
```

Fragment referenced in 45.

Uses: B 27a, C\_get\_B 146a, C\_get\_dimTable 145d, C\_get\_ExpectationInfluence 144b, C\_get\_ExpectationX 144a, C\_get\_P 141c, C\_get\_Q 142a, C\_get\_Sumweights 145b, C\_get\_Table 145c, C\_get\_varonly 142b, C\_get\_Xfactor 142c, P 24a, PP12 131b, Q 24e, sumweights 25e.

```
> LinStatExpCov(X = iX2d, ix = ix, Y = iY2d, iy = iy,
+               weights = weights, subset = subset, nresample = 10)$PermutedLinearStatistic

      [,1]      [,2]      [,3]      [,4]      [,5]      [,6]      [,7]
[1,] 15.486226 15.432786 15.474636 15.434733 15.515989 15.421226 15.523577
[2,]  7.864472  7.948006  8.105228  8.390763  8.212044  8.493673  8.415919
[3,] 12.398382 12.290212 11.905712 12.506639 12.143855 12.549147 12.590900
[4,] 31.244688 31.476627 31.257920 31.264541 31.096744 31.405390 31.259421
[5,] 17.500047 17.688850 17.055915 15.065147 16.586396 15.315949 16.382058
[6,] 24.466142 24.647923 25.464893 24.239801 25.488434 24.296588 23.694321
[7,] 43.423057 43.421097 43.330443 43.612924 43.424099 43.430492 43.309780
[8,] 24.311651 23.474319 22.844531 23.490053 23.541204 22.749540 22.388328
[9,] 34.180046 34.430423 35.397534 33.988759 34.366957 33.293748 33.389741
[10,] 13.461330 13.490553 13.492064 13.434007 13.447127 13.491634 13.476994
[11,]  6.973432  7.169633  7.259611  6.943487  7.017767  7.094398  7.241183
[12,] 10.672723 10.658055 10.574382 10.675107 10.743783 10.837748 10.788257

      [,8]      [,9]      [,10]
[1,] 15.434192 15.491818 15.398248
[2,]  7.834800  8.223809  7.925817
[3,] 12.362877 11.997518 12.169918
[4,] 31.510352 31.284964 31.576643
[5,] 18.211173 16.969768 17.197270
[6,] 23.773081 25.183959 24.742788
[7,] 43.375471 43.374905 43.496870
```

```
[8,] 23.445718 22.372659 23.729797
[9,] 34.264857 35.341197 34.487409
[10,] 13.498456 13.473376 13.482370
[11,] 7.221054 7.329256 7.097392
[12,] 10.669965 10.540119 10.702889
```

`<R_PermutatedLinearStatistic_2d Prototype 47a> ≡`

```
SEXP R_PermutatedLinearStatistic_2d
(
  <R x Input 23d>
  SEXP ix,
  <R y Input 24d>
  SEXP iy,
  <R block Input 26f>,
  SEXP nresample,
  SEXP itable
)
◇
```

Fragment referenced in [22b](#), [48](#).

Uses: [R\\_PermutatedLinearStatistic\\_2d 48](#).

This function can be called from other packages.

`"libcoinAPI.h" 47b ≡`

```
extern SEXP libcoin_R_PermutatedLinearStatistic_2d(
  SEXP x, SEXP ix, SEXP y, SEXP iy, SEXP block, SEXP nresample,
  SEXP itable
) {
  static SEXP(*fun)(SEXP, SEXP, SEXP, SEXP, SEXP, SEXP, SEXP) = NULL;
  if (fun == NULL)
    fun = (SEXP*)(SEXP, SEXP, SEXP, SEXP, SEXP, SEXP, SEXP)
      R_GetCCallable("libcoin", "R_PermutatedLinearStatistic_2d");
  return fun(x, ix, y, iy, block, nresample, itable);
}
◇
```

File defined by [30d](#), [36c](#), [38c](#), [40b](#), [47b](#), [50c](#), [53](#), [55b](#), [61a](#), [132c](#), [136a](#), [138b](#), [140b](#).

Uses: [block 26f](#), [27b](#), [R\\_PermutatedLinearStatistic\\_2d 48](#), [x 23d](#), [24bc](#), [y 24df](#), [25a](#).

*< R\_PermutedLinearStatistic\_2d 48 >* ≡

*< R\_PermutedLinearStatistic\_2d Prototype 47a >*

```
{
  SEXP ans, Ritable;
  int *csum, *rsum, *sumweights, *jwork, *table, *rtable2, maxn = 0, Lxp1, Lyp1, *btab, PQ, Xfactor;
  R_xlen_t inresample;
  double *fact, *linstat;

  < Setup Dimensions 2d 41b >

  PQ = mPQB(P, Q, 1);
  Xfactor = XLENGTH(x) == 0;
  Lxp1 = Lx + 1;
  Lyp1 = Ly + 1;
  inresample = (R_xlen_t) REAL(nresample)[0];

  PROTECT(ans = allocMatrix(REALSXP, PQ, inresample));

  < Setup Working Memory 49b >

  < Convert Table to Integer 49a >

  for (int b = 0; b < B; b++) {
    btab = INTEGER(Ritable) + Lxp1 * Lyp1 * b;
    < Col Row Total Sums 43a >
    if (sumweights[b] > maxn) maxn = sumweights[b];
  }

  < Setup Log-Factorials 49c >

  GetRNGstate();

  for (R_xlen_t np = 0; np < inresample; np++) {
    < Setup Linear Statistic 38a >

    for (int p = 0; p < Lxp1 * Lyp1; p++)
      table[p] = 0;

    for (int b = 0; b < B; b++) {
      < Compute Permuted Linear Statistic 2d 49d >
    }
  }

  PutRNGstate();

  R_Free(csum); R_Free(rsum); R_Free(sumweights); R_Free(rtable2);
  R_Free(jwork); R_Free(fact); R_Free(table);
  UNPROTECT(2);
  return(ans);
}
◇
```

Fragment referenced in [39b](#).

Defines: [R\\_PermutedLinearStatistic\\_2d 8](#), [47ab](#), [49a](#), [152c](#), [153](#).

Uses: [B 27a](#), [mPQB 132a](#), [P 24a](#), [Q 24e](#), [sumweights 25e](#), [x 23d](#), [24bc](#).

⟨ *Convert Table to Integer 49a* ⟩ ≡

```
PROTECT(Ritable = allocVector(INTSXP, LENGTH(itable)));
for (int i = 0; i < LENGTH(itable); i++) {
  if (REAL(itable)[i] > INT_MAX)
    error("cannot deal with weights larger INT_MAX in R_PermedLinearStatistic_2d");
  INTEGER(Ritable)[i] = (int) REAL(itable)[i];
}
◇
```

Fragment referenced in [48](#).

Uses: [R\\_PermedLinearStatistic\\_2d 48](#), [weights 25b](#).

⟨ *Setup Working Memory 49b* ⟩ ≡

```
csum = R_Calloc(Lyp1 * B, int);
rsum = R_Calloc(Lxp1 * B, int);
sumweights = R_Calloc(B, int);
table = R_Calloc(Lxp1 * Lyp1, int);
rtable2 = R_Calloc(Lx * Ly, int);
jwork = R_Calloc(Lyp1, int);
◇
```

Fragment referenced in [48](#).

Uses: [B 27a](#), [sumweights 25e](#).

⟨ *Setup Log-Factorials 49c* ⟩ ≡

```
fact = R_Calloc(maxn + 1, double);
/* Calculate log-factorials. fact[i] = lgamma(i+1) */
fact[0] = fact[1] = 0.;
for (int j = 2; j <= maxn; j++)
  fact[j] = fact[j - 1] + log(j);
◇
```

Fragment referenced in [48](#).

Note: the interface to `S_rcont2` changed in R-4.1.0.

⟨ *Compute Permuted Linear Statistic 2d 49d* ⟩ ≡

```
#if defined(R_VERSION) && R_VERSION >= R_Version(4, 1, 0)
  S_rcont2(Lx, Ly,
           rsum + Lxp1 * b + 1,
           csum + Lyp1 * b + 1,
           sumweights[b], fact, jwork, rtable2);
#else
  S_rcont2(&Lx, &Ly,
           rsum + Lxp1 * b + 1,
           csum + Lyp1 * b + 1,
           sumweights + b, fact, jwork, rtable2);
#endif

for (int j1 = 1; j1 <= Lx; j1++) {
  for (int j2 = 1; j2 <= Ly; j2++)
    table[j2 * Lxp1 + j1] = rtable2[(j2 - 1) * Lx + (j1 - 1)];
}
btab = table;
⟨ Linear Statistic 2d 42a ⟩
◇
```

Fragment referenced in [48](#).

Uses: [sumweights 25e](#).

## 3.5 Tests

*< Tests 50a >* ≡

```
< R_QuadraticTest 51 >
< R_MaximumTest 54 >
< R_MaximallySelectedTest 56 >
◇
```

Fragment referenced in [23a](#).

*< R\_QuadraticTest Prototype 50b >* ≡

```
SEXP R_QuadraticTest
(
  < R_LECV Input 141b >,
  SEXP pvalue,
  SEXP lower,
  SEXP give_log,
  SEXP PermutedStatistics
)
◇
```

Fragment referenced in [22b](#), [51](#).

This function can be called from other packages.

"libcoinAPI.h" 50c ≡

```
extern SEXP libcoin_R_QuadraticTest(
  SEXP LECV, SEXP pvalue, SEXP lower, SEXP give_log, SEXP PermutedStatistics
) {
  static SEXP(*fun)(SEXP, SEXP, SEXP, SEXP, SEXP) = NULL;
  if (fun == NULL)
    fun = (SEXP*)(SEXP, SEXP, SEXP, SEXP, SEXP)
      R_GetCCallable("libcoin", "R_QuadraticTest");
  return fun(LECV, pvalue, lower, give_log, PermutedStatistics);
}
◇
```

File defined by [30d](#), [36c](#), [38c](#), [40b](#), [47b](#), [50c](#), [53](#), [55b](#), [61a](#), [132c](#), [136a](#), [138b](#), [140b](#).  
Uses: [LECV 141b](#).

*< R\_QuadraticTest 51 >* ≡

*< R\_QuadraticTest Prototype 50b >*

```
{
  SEXP ans, stat, pval, names, permstat;
  double *MPinv, *ls, st, pst, *ex;
  int rank, P, Q, PQ, greater = 0;
  R_xlen_t nresample;

  < Setup Test Memory 52a >

  MPinv = R_Calloc(PP12(PQ), double); /* was: C_get_MPinv(LECV); */
  C_MPinv_sym(C_get_Covariance(LECV), PQ, C_get_tol(LECV), MPinv, &rank);

  REAL(stat)[0] = C_quadform(PQ, C_get_LinearStatistic(LECV),
                             C_get_Expectation(LECV), MPinv);

  if (!PVALUE) {
    UNPROTECT(2);
    R_Free(MPinv);
    return(ans);
  }

  if (C_get_nresample(LECV) == 0) {
    REAL(pval)[0] = C_chisq_pvalue(REAL(stat)[0], rank, LOWER, GIVELOG);
  } else {
    nresample = C_get_nresample(LECV);
    ls = C_get_PermutedLinearStatistic(LECV);
    st = REAL(stat)[0];
    ex = C_get_Expectation(LECV);
    greater = 0;
    for (R_xlen_t np = 0; np < nresample; np++) {
      pst = C_quadform(PQ, ls + PQ * np, ex, MPinv);
      if (GE(pst, st, C_get_tol(LECV)))
        greater++;
      if (PSTAT) REAL(permstat)[np] = pst;
    }
    REAL(pval)[0] = C_perm_pvalue(greater, nresample, LOWER, GIVELOG);
  }

  UNPROTECT(2);
  R_Free(MPinv);
  return(ans);
}
◇
```

Fragment referenced in [50a](#).

Uses: [C\\_chisq\\_pvalue 64c](#), [C\\_get\\_Covariance 143c](#), [C\\_get\\_Expectation 143a](#), [C\\_get\\_LinearStatistic 142d](#),  
[C\\_get\\_nresample 146b](#), [C\\_get\\_PermutedLinearStatistic 146c](#), [C\\_get\\_tol 146d](#), [C\\_perm\\_pvalue 65](#), [C\\_quadform 62](#),  
[GE 21a](#), [LECV 141b](#), [P 24a](#), [PP12 131b](#), [Q 24e](#).

*Setup Test Memory 52a* ≡

```
P = C_get_P(LECV);
Q = C_get_Q(LECV);
PQ = mPQB(P, Q, 1);

if (C_get_varonly(LECV) && PQ > 1)
  error("cannot compute adjusted p-value based on variances only");
/* if (C_get_nresample(LECV) > 0 && INTEGER(PermutedStatistics)[0]) { */
  PROTECT(ans = allocVector(VECSXP, 3));
  PROTECT(names = allocVector(STRSXP, 3));
  SET_VECTOR_ELT(ans, 2, permstat = allocVector(REALSXP, C_get_nresample(LECV)));
  SET_STRING_ELT(names, 2, mkChar("PermutedStatistics"));
/* } else {
  PROTECT(ans = allocVector(VECSXP, 2));
  PROTECT(names = allocVector(STRSXP, 2));
}
*/
SET_VECTOR_ELT(ans, 0, stat = allocVector(REALSXP, 1));
SET_STRING_ELT(names, 0, mkChar("TestStatistic"));
SET_VECTOR_ELT(ans, 1, pval = allocVector(REALSXP, 1));
SET_STRING_ELT(names, 1, mkChar("p.value"));
namesgets(ans, names);
REAL(pval)[0] = NA_REAL;
int LOWER = INTEGER(lower)[0];
int GIVELOG = INTEGER(give_log)[0];
int PVALUE = INTEGER(pvalue)[0];
int PSTAT = INTEGER(PermutedStatistics)[0];
◇
```

Fragment referenced in [51](#), [54](#).

Uses: [C\\_get\\_nresample 146b](#), [C\\_get\\_P 141c](#), [C\\_get\\_Q 142a](#), [C\\_get\\_varonly 142b](#), [LECV 141b](#), [mPQB 132a](#), [P 24a](#), [Q 24e](#).

*R\_MaximumTest Prototype 52b* ≡

```
SEXP R_MaximumTest
(
  R LECV Input 141b,
  SEXP alternative,
  SEXP pvalue,
  SEXP lower,
  SEXP give_log,
  SEXP PermutedStatistics,
  SEXP maxpts,
  SEXP releps,
  SEXP abseps
)
◇
```

Fragment referenced in [22b](#), [54](#).

This function can be called from other packages.

"libcoinAPI.h" 53≡

```
extern SEXP libcoin_R_MaximumTest(  
  SEXP LECV, SEXP alternative, SEXP pvalue, SEXP lower, SEXP give_log,  
  SEXP PermutedStatistics, SEXP maxpts, SEXP releps, SEXP abseps  
) {  
  static SEXP(*fun)(SEXP, SEXP, SEXP, SEXP, SEXP, SEXP, SEXP, SEXP, SEXP) = NULL;  
  if (fun == NULL)  
    fun = (SEXP*)(SEXP, SEXP, SEXP, SEXP, SEXP, SEXP, SEXP, SEXP, SEXP)  
      R_GetCCallable("libcoin", "R_MaximumTest");  
  return fun(LECV, alternative, pvalue, lower, give_log, PermutedStatistics, maxpts, releps,  
    abseps);  
}  
◇
```

File defined by [30d](#), [36c](#), [38c](#), [40b](#), [47b](#), [50c](#), [53](#), [55b](#), [61a](#), [132c](#), [136a](#), [138b](#), [140b](#).  
Uses: [LECV 141b](#).

*< R\_MaximumTest 54 >* ≡

```
< R_MaximumTest Prototype 52b >
{
  SEXP ans, stat, pval, names, permstat;
  double st, pst, *ex, *cv, *ls, tl;
  int P, Q, PQ, vo, alt, greater;
  R_xlen_t nresample;

  < Setup Test Memory 52a >

  if (C_get_varonly(LECV)) {
    cv = C_get_Variance(LECV);
  } else {
    cv = C_get_Covariance(LECV);
  }
  REAL(stat)[0] = C_maxtype(PQ, C_get_LinearStatistic(LECV),
    C_get_Expectation(LECV), cv, C_get_varonly(LECV), C_get_tol(LECV),
    INTEGER(alternative)[0]);
  if (!PVALUE) {
    UNPROTECT(2);
    return(ans);
  }

  if (C_get_nresample(LECV) == 0) {
    if (C_get_varonly(LECV) && PQ > 1) {
      REAL(pval)[0] = NA_REAL;
      UNPROTECT(2);
      return(ans);
    }
    REAL(pval)[0] = C_maxtype_pvalue(REAL(stat)[0], cv,
      PQ, INTEGER(alternative)[0], LOWER, GIVELOG, INTEGER(maxpts)[0],
      REAL(releps)[0], REAL(abseps)[0], C_get_tol(LECV));
  } else {
    nresample = C_get_nresample(LECV);
    ls = C_get_PermutedLinearStatistic(LECV);
    ex = C_get_Expectation(LECV);
    vo = C_get_varonly(LECV);
    alt = INTEGER(alternative)[0];
    st = REAL(stat)[0];
    tl = C_get_tol(LECV);
    greater = 0;
    for (R_xlen_t np = 0; np < nresample; np++) {
      pst = C_maxtype(PQ, ls + PQ * np, ex, cv, vo, tl, alt);
      if (alt == ALTERNATIVE_less) {
        if (LE(pst, st, tl)) greater++;
      } else {
        if (GE(pst, st, tl)) greater++;
      }
      if (PSTAT) REAL(permstat)[np] = pst;
    }
    REAL(pval)[0] = C_perm_pvalue(greater, nresample, LOWER, GIVELOG);
  }
  UNPROTECT(2);
  return(ans);
}
◇
```

Fragment referenced in 50a.

Uses: C\_get\_Covariance 143c, C\_get\_Expectation 143a, C\_get\_LinearStatistic 142d, C\_get\_nresample 146b, C\_get\_tol 146d, C\_get\_Variance 143b, C\_get\_varonly 142b, C\_maxtype 63, C\_maxtype\_pvalue 67, C\_perm\_pvalue 65, GE 21a, LE 21a, LECV 141b, P 24a, Q 24e.

*< R\_MaximallySelectedTest Prototype 55a >* ≡

```
SEXP R_MaximallySelectedTest
(
    SEXP LECV,
    SEXP ordered,
    SEXP teststat,
    SEXP minbucket,
    SEXP lower,
    SEXP give_log
)
◇
```

Fragment referenced in [22b](#), [56](#).  
Uses: [LECV 141b](#).

This function can be called from other packages.

"libcoinAPI.h" 55b≡

```
extern SEXP libcoin_R_MaximallySelectedTest(
    SEXP LECV, SEXP ordered, SEXP teststat, SEXP minbucket, SEXP lower, SEXP give_log
) {
    static SEXP(*fun)(SEXP, SEXP, SEXP, SEXP, SEXP, SEXP) = NULL;
    if (fun == NULL)
        fun = (SEXP*)(SEXP, SEXP, SEXP, SEXP, SEXP, SEXP)
            R_GetCCallable("libcoin", "R_MaximallySelectedTest");
    return fun(LECV, ordered, teststat, minbucket, lower, give_log);
}
◇
```

File defined by [30d](#), [36c](#), [38c](#), [40b](#), [47b](#), [50c](#), [53](#), [55b](#), [61a](#), [132c](#), [136a](#), [138b](#), [140b](#).  
Uses: [LECV 141b](#).

*< R\_MaximallySelectedTest 56 >* ≡

*< R\_MaximallySelectedTest Prototype 55a >*

```
{
  SEXP ans, index, stat, pval, names, permstat;
  int P, mb;

  P = C_get_P(LECV);
  mb = INTEGER(minbucket)[0];

  PROTECT(ans = allocVector(VECSXP, 4));
  PROTECT(names = allocVector(STRSXP, 4));
  SET_VECTOR_ELT(ans, 0, stat = allocVector(REALSXP, 1));
  SET_STRING_ELT(names, 0, mkChar("TestStatistic"));
  SET_VECTOR_ELT(ans, 1, pval = allocVector(REALSXP, 1));
  SET_STRING_ELT(names, 1, mkChar("p.value"));
  SET_VECTOR_ELT(ans, 3, permstat = allocVector(REALSXP, C_get_nresample(LECV)));
  SET_STRING_ELT(names, 3, mkChar("PermutedStatistics"));
  REAL(pval)[0] = NA_REAL;

  if (INTEGER(ordered)[0]) {
    SET_VECTOR_ELT(ans, 2, index = allocVector(INTSXP, 1));
    C_ordered_Xfactor(LECV, mb, INTEGER(teststat)[0],
                     INTEGER(index), REAL(stat), REAL(permstat),
                     REAL(pval), INTEGER(lower)[0],
                     INTEGER(give_log)[0]);
    if (REAL(stat)[0] > 0)
      INTEGER(index)[0]++; /* R style indexing */
  } else {
    SET_VECTOR_ELT(ans, 2, index = allocVector(INTSXP, P));
    C_unordered_Xfactor(LECV, mb, INTEGER(teststat)[0],
                       INTEGER(index), REAL(stat), REAL(permstat),
                       REAL(pval), INTEGER(lower)[0],
                       INTEGER(give_log)[0]);
  }

  SET_STRING_ELT(names, 2, mkChar("index"));
  namesgets(ans, names);

  UNPROTECT(2);
  return(ans);
}
◇
```

Fragment referenced in [50a](#).

Uses: [C\\_get\\_nresample 146b](#), [C\\_get\\_P 141c](#), [C\\_ordered\\_Xfactor 70](#), [C\\_unordered\\_Xfactor 74](#), [LECV 141b](#), [P 24a](#).

## 3.6 Test Statistics

⟨ *Test Statistics 57a* ⟩ ≡

⟨ *C\_maxstand\_Covariance 57b* ⟩  
⟨ *C\_maxstand\_Variance 58a* ⟩  
⟨ *C\_minstand\_Covariance 58b* ⟩  
⟨ *C\_minstand\_Variance 59a* ⟩  
⟨ *C\_maxabsstand\_Covariance 59b* ⟩  
⟨ *C\_maxabsstand\_Variance 60a* ⟩  
⟨ *C\_quadform 62* ⟩  
⟨ *R\_quadform 61b* ⟩  
⟨ *C\_maxtype 63* ⟩  
⟨ *C\_standardise 64a* ⟩  
⟨ *C\_ordered\_Xfactor 70* ⟩  
⟨ *C\_unordered\_Xfactor 74* ⟩  
◇

Fragment referenced in [23a](#).

⟨ *C\_maxstand\_Covariance 57b* ⟩ ≡

```
double C_maxstand_Covariance
(
    const int PQ,
    const double *linstat,
    const double *expect,
    const double *covar_sym,
    const double tol
) {
    double ans = R_NegInf, tmp = 0.0;

    for (int p = 0; p < PQ; p++) {
        tmp = 0.0;
        if (covar_sym[S(p, p, PQ)] > tol)
            tmp = (linstat[p] - expect[p]) / sqrt(covar_sym[S(p, p, PQ)]);
        if (tmp > ans) ans = tmp;
    }
    return(ans);
}
◇
```

Fragment referenced in [57a](#).

Defines: [C\\_maxstand\\_Covariance 63](#).

Uses: [S 21a](#).

$\langle C\_maxstand\_Variance\ 58a \rangle \equiv$

```
double C_maxstand_Variance
(
    const int PQ,
    const double *linstat,
    const double *expect,
    const double *var,
    const double tol
) {
    double ans = R_NegInf, tmp = 0.0;

    for (int p = 0; p < PQ; p++) {
        tmp = 0.0;
        if (var[p] > tol)
            tmp = (linstat[p] - expect[p]) / sqrt(var[p]);
        if (tmp > ans) ans = tmp;
    }
    return(ans);
}
◇
```

Fragment referenced in [57a](#).

Defines: [C\\_maxstand\\_Variance 63](#).

$\langle C\_minstand\_Covariance\ 58b \rangle \equiv$

```
double C_minstand_Covariance
(
    const int PQ,
    const double *linstat,
    const double *expect,
    const double *covar_sym,
    const double tol
) {
    double ans = R_PosInf, tmp = 0.0;

    for (int p = 0; p < PQ; p++) {
        tmp = 0.0;
        if (covar_sym[S(p, p, PQ)] > tol)
            tmp = (linstat[p] - expect[p]) / sqrt(covar_sym[S(p, p, PQ)]);
        if (tmp < ans) ans = tmp;
    }
    return(ans);
}
◇
```

Fragment referenced in [57a](#).

Defines: [C\\_minstand\\_Covariance 63](#).

Uses: [S 21a](#).

$\langle C\_minstand\_Variance\ 59a \rangle \equiv$

```
double C_minstand_Variance
(
    const int PQ,
    const double *linstat,
    const double *expect,
    const double *var,
    const double tol
) {
    double ans = R_PosInf, tmp = 0.0;

    for (int p = 0; p < PQ; p++) {
        tmp = 0.0;
        if (var[p] > tol)
            tmp = (linstat[p] - expect[p]) / sqrt(var[p]);
        if (tmp < ans) ans = tmp;
    }
    return(ans);
}
◇
```

Fragment referenced in [57a](#).

Defines: `C_minstand_Variance` [63](#).

$\langle C\_maxabsstand\_Covariance\ 59b \rangle \equiv$

```
double C_maxabsstand_Covariance
(
    const int PQ,
    const double *linstat,
    const double *expect,
    const double *covar_sym,
    const double tol
) {
    double ans = R_NegInf, tmp = 0.0;

    for (int p = 0; p < PQ; p++) {
        tmp = 0.0;
        if (covar_sym[S(p, p, PQ)] > tol)
            tmp = fabs((linstat[p] - expect[p]) /
                sqrt(covar_sym[S(p, p, PQ)]));
        if (tmp > ans) ans = tmp;
    }
    return(ans);
}
◇
```

Fragment referenced in [57a](#).

Defines: `C_maxabsstand_Covariance` [63](#).

Uses: `S` [21a](#).

*< C\_maxabsstand\_Variance 60a >* ≡

```
double C_maxabsstand_Variance
(
  const int PQ,
  const double *linstat,
  const double *expect,
  const double *var,
  const double tol
) {
  double ans = R_NegInf, tmp = 0.0;

  for (int p = 0; p < PQ; p++) {
    tmp = 0.0;
    if (var[p] > tol)
      tmp = fabs((linstat[p] - expect[p]) / sqrt(var[p]));
    if (tmp > ans) ans = tmp;
  }
  return(ans);
}
◇
```

Fragment referenced in [57a](#).

Defines: `C_maxabsstand_Variance` [63](#).

```
> MPinverse <-
+ function(x, tol = sqrt(.Machine$double.eps))
+ {
+   SVD <- svd(x)
+   pos <- SVD$d > max(tol * SVD$d[1L], 0)
+   inv <- SVD$v[, pos, drop = FALSE] %%
+     ((1/SVD$d[pos]) * t(SVD$u[, pos, drop = FALSE]))
+   list(MPinv = inv, rank = sum(pos))
+ }
> quadform <-
+ function(linstat, expect, MPinv)
+ {
+   censtat <- linstat - expect
+   censtat %% MPinv %% censtat
+ }
> linstat <- ls1$LinearStatistic
> expect <- ls1$Expectation
> MPinv <- MPinverse(vcov(ls1))$MPinv
> MPinv_sym <- MPinv[lower.tri(MPinv, diag = TRUE)]
> qf1 <- quadform(linstat, expect, MPinv)
> qf2 <- .Call(libcoin:::R_quadform, linstat, expect, MPinv_sym)
> stopifnot(isequal(qf1, qf2))
```

*< R\_quadform Prototype 60b >* ≡

```
SEXP R_quadform
(
  SEXP linstat,
  SEXP expect,
  SEXP MPinv_sym
)
◇
```

Fragment referenced in [22b](#), [61b](#).

Uses: `R_quadform` [61b](#).

This function can be called from other packages.

"libcoinAPI.h" 61a≡

```
extern SEXP libcoin_R_quadform(  
    SEXP linstat, SEXP expect, SEXP MPinv_sym  
) {  
    static SEXP(*fun)(SEXP, SEXP, SEXP) = NULL;  
    if (fun == NULL)  
        fun = (SEXP*)(SEXP, SEXP, SEXP)  
            R_GetCCallable("libcoin", "R_quadform");  
    return fun(linstat, expect, MPinv_sym);  
}  
◇
```

File defined by 30d, 36c, 38c, 40b, 47b, 50c, 53, 55b, 61a, 132c, 136a, 138b, 140b.  
Uses: R\_quadform 61b.

⟨ R\_quadform 61b ⟩ ≡

```
⟨ R_quadform Prototype 60b ⟩  
{  
    SEXP ans;  
    int n, PQ;  
    double *dlinstat, *dexpect, *dMPinv_sym, *dans;  
  
    n = NCOL(linstat);  
    PQ = NROW(linstat);  
    dlinstat = REAL(linstat);  
    dexpect = REAL(expect);  
    dMPinv_sym = REAL(MPinv_sym);  
  
    PROTECT(ans = allocVector(REALSXP, n));  
    dans = REAL(ans);  
    for (int i = 0; i < n; i++)  
        dans[i] = C_quadform(PQ, dlinstat + PQ * i, dexpect, dMPinv_sym);  
  
    UNPROTECT(1);  
    return(ans);  
}  
◇
```

Fragment referenced in 57a.

Defines: R\_quadform 60b, 61a, 152c, 153.

Uses: C\_quadform 62, NCOL 130c, NROW 130b.

$\langle C\_quadform\ 62 \rangle \equiv$

```
double C_quadform
(
    const int PQ,
    const double *linstat,
    const double *expect,
    const double *MPinv_sym
) {
    double ans = 0.0, tmp = 0.0;

    for (int q = 0; q < PQ; q++) {
        tmp = 0.0;
        for (int p = 0; p < PQ; p++)
            tmp += (linstat[p] - expect[p]) * MPinv_sym[S(p, q, PQ)];
        ans += tmp * (linstat[q] - expect[q]);
    }

    return(ans);
}
◇
```

Fragment referenced in [57a](#).

Defines: [C\\_quadform 51](#), [61b](#), [73c](#).

Uses: [S 21a](#).

$\langle C\_maxtype\ 63 \rangle \equiv$

```
double C_maxtype
(
    const int PQ,
    const double *linstat,
    const double *expect,
    const double *covar,
    const int varonly,
    const double tol,
    const int alternative
) {
    double ret = 0.0;

    if (varonly) {
        if (alternative == ALTERNATIVE_twosided) {
            ret = C_maxabsstand_Variance(PQ, linstat, expect, covar, tol);
        } else if (alternative == ALTERNATIVE_less) {
            ret = C_minstand_Variance(PQ, linstat, expect, covar, tol);
        } else if (alternative == ALTERNATIVE_greater) {
            ret = C_maxstand_Variance(PQ, linstat, expect, covar, tol);
        }
    } else {
        if (alternative == ALTERNATIVE_twosided) {
            ret = C_maxabsstand_Covariance(PQ, linstat, expect, covar, tol);
        } else if (alternative == ALTERNATIVE_less) {
            ret = C_minstand_Covariance(PQ, linstat, expect, covar, tol);
        } else if (alternative == ALTERNATIVE_greater) {
            ret = C_maxstand_Covariance(PQ, linstat, expect, covar, tol);
        }
    }
    return(ret);
}
◇
```

Fragment referenced in [57a](#).

Defines: [C\\_maxtype 54](#), [73c](#).

Uses: [C\\_maxabsstand\\_Covariance 59b](#), [C\\_maxabsstand\\_Variance 60a](#), [C\\_maxstand\\_Covariance 57b](#),  
[C\\_maxstand\\_Variance 58a](#), [C\\_minstand\\_Covariance 58b](#), [C\\_minstand\\_Variance 59a](#).

$\langle C\_standardise\ 64a \rangle \equiv$

```
void C_standardise
(
    const int PQ,
    double *linstat,      /* in place standardisation */
    const double *expect,
    const double *covar,
    const int varonly,
    const double tol
) {
    double var;

    for (int p = 0; p < PQ; p++) {
        if (varonly) {
            var = covar[p];
        } else {
            var = covar[S(p, p, PQ)];
        }
        if (var > tol) {
            linstat[p] = (linstat[p] - expect[p]) / sqrt(var);
        } else {
            linstat[p] = NAN;
        }
    }
}
```

Fragment referenced in [57a](#).  
Defines: [C\\_standardise 39a](#).  
Uses: [S 21a](#).

$\langle P\text{-Values}\ 64b \rangle \equiv$

```
 $\langle C\_chisq\_pvalue\ 64c \rangle$   
 $\langle C\_perm\_pvalue\ 65 \rangle$   
 $\langle C\_norm\_pvalue\ 66 \rangle$   
 $\langle C\_maxtype\_pvalue\ 67 \rangle$   
◇
```

Fragment referenced in [23a](#).

$\langle C\_chisq\_pvalue\ 64c \rangle \equiv$

```
/* lower = 1 means p-value, lower = 0 means 1 - p-value */
double C_chisq_pvalue
(
    const double stat,
    const int df,
    const int lower,
    const int give_log
) {
    return(pchisq(stat, (double) df, lower, give_log));
}
```

Fragment referenced in [64b](#).  
Defines: [C\\_chisq\\_pvalue 51](#).

$\langle C\_perm\_pvalue\ 65 \rangle \equiv$

```
double C_perm_pvalue
(
    const int greater,
    const double nresample,
    const int lower,
    const int give_log
) {
    double ret;

    if (give_log) {
        if (lower) {
            ret = log1p(- (double) greater / nresample);
        } else {
            ret = log(greater) - log(nresample);
        }
    } else {
        if (lower) {
            ret = 1.0 - (double) greater / nresample;
        } else {
            ret = (double) greater / nresample;
        }
    }
    return(ret);
}
◇
```

Fragment referenced in [64b](#).

Defines: `C_perm_pvalue` [51](#), [54](#), [73d](#).

$\langle C\_norm\_pvalue\ 66 \rangle \equiv$

```
double C_norm_pvalue
(
    const double stat,
    const int alternative,
    const int lower,
    const int give_log
) {
    double ret;

    if (alternative == ALTERNATIVE_less) {
        return(pnorm(stat, 0.0, 1.0, 1 - lower, give_log));
    } else if (alternative == ALTERNATIVE_greater) {
        return(pnorm(stat, 0.0, 1.0, lower, give_log));
    } else if (alternative == ALTERNATIVE_twosided) {
        if (lower) {
            ret = pnorm(fabs(stat)*-1.0, 0.0, 1.0, 1, 0);
            if (give_log) {
                return(log1p(- 2 * ret));
            } else {
                return(1 - 2 * ret);
            }
        } else {
            ret = pnorm(fabs(stat)*-1.0, 0.0, 1.0, 1, give_log);
            if (give_log) {
                return(ret + log(2));
            } else {
                return(2 * ret);
            }
        }
    }
    return(NA_REAL);
}
◇
```

Fragment referenced in [64b](#).

$\langle C\_maxtype\_pvalue\ 67 \rangle \equiv$

```
double C_maxtype_pvalue
(
    const double stat,
    const double *Covariance,
    const int n,
    const int alternative,
    const int lower,
    const int give_log,
    int maxpts, /* const? */
    double releps,
    double abseps,
    double tol
) {
    int nu = 0, inform, i, j, sub, nonzero, *infin, *index, rnd = 0;
    double ans, myerror, *lowerbnd, *upperbnd, *delta, *corr, *sd;

    /* univariate problem */
    if (n == 1)
        return(C_norm_pvalue(stat, alternative, lower, give_log));

     $\langle Setup\ mvtnorm\ Memory\ 68 \rangle$ 

     $\langle Setup\ mvtnorm\ Correlation\ 69a \rangle$ 

    /* call mvtnorm's mvtdst C function defined in mvtnorm/include/mvtnormAPI.h */
    mvtnorm_C_mvtdst(&nonzero, &nu, lowerbnd, upperbnd, infin, corr, delta,
        &maxpts, &abseps, &releps, &myerror, &ans, &inform, &rnd);

    /* inform == 0 means: everything is OK */
    switch (inform) {
        case 0: break;
        case 1: warning("cmvnorm: completion with ERROR > EPS"); break;
        case 2: warning("cmvnorm: N > 1000 or N < 1");
            ans = 0.0;
            break;
        case 3: warning("cmvnorm: correlation matrix not positive semi-definite");
            ans = 0.0;
            break;
        default: warning("cmvnorm: unknown problem in MVT DST");
            ans = 0.0;
    }
    R_Free(corr); R_Free(sd); R_Free(lowerbnd); R_Free(upperbnd);
    R_Free(infin); R_Free(delta); R_Free(index);

    /* ans = 1 - p-value */
    if (lower) {
        if (give_log)
            return(log(ans)); /* log(1 - p-value) */
        return(ans); /* 1 - p-value */
    } else {
        if (give_log)
            return(log1p(ans)); /* log(p-value) */
        return(1 - ans); /* p-value */
    }
}
◇
```

Fragment referenced in [64b](#).  
Defines: `C_maxtype_pvalue` [54](#).  
Uses: [N 23bc](#).

*Setup mvtnorm Memory 68* ≡

```
if (n == 2)
  corr = R_Calloc(1, double);
else
  corr = R_Calloc(n + ((n - 2) * (n - 1))/2, double);

sd = R_Calloc(n, double);
lowerbnd = R_Calloc(n, double);
upperbnd = R_Calloc(n, double);
infin = R_Calloc(n, int);
delta = R_Calloc(n, double);
index = R_Calloc(n, int);

/* determine elements with non-zero variance */

nonzero = 0;
for (i = 0; i < n; i++) {
  if (Covariance[S(i, i, n)] > tol) {
    index[nonzero] = i;
    nonzero++;
  }
}
◇
```

Fragment referenced in [67](#).

Uses: [S 21a](#).

`mvtdst` assumes the unique elements of the triangular covariance matrix to be passed as argument `CORREL`

*Setup mvtnorm Correlation 69a* ≡

```
for (int nz = 0; nz < nonzero; nz++) {
  /* handle elements with non-zero variance only */
  i = index[nz];

  /* standard deviations */
  sd[i] = sqrt(Covariance[S(i, i, n)]);

  if (alternative == ALTERNATIVE_less) {
    lowerbnd[nz] = stat;
    upperbnd[nz] = R_PosInf;
    infin[nz] = 1;
  } else if (alternative == ALTERNATIVE_greater) {
    lowerbnd[nz] = R_NegInf;
    upperbnd[nz] = stat;
    infin[nz] = 0;
  } else if (alternative == ALTERNATIVE_twosided) {
    lowerbnd[nz] = fabs(stat) * -1.0;
    upperbnd[nz] = fabs(stat);
    infin[nz] = 2;
  }

  delta[nz] = 0.0;

  /* set up vector of correlations, i.e., the upper
  triangular part of the covariance matrix) */
  for (int jz = 0; jz < nz; jz++) {
    j = index[jz];
    sub = (int) (jz + 1) + (double) ((nz - 1) * nz) / 2 - 1;
    if (sd[i] == 0.0 || sd[j] == 0.0)
      corr[sub] = 0.0;
    else
      corr[sub] = Covariance[S(i, j, n)] / (sd[i] * sd[j]);
  }
}
◇
```

Fragment referenced in [67](#).  
Uses: [S 21a](#).

*maxstat Xfactor Variables 69b* ≡

```
SEXP LECV,
const int minbucket,
const int teststat,
int *wmax,
double *maxstat,
double *bmaxstat,
double *pval,
const int lower,
const int give_log
◇
```

Fragment referenced in [70](#), [74](#).  
Uses: [LECV 141b](#).

*< C\_ordered\_Xfactor 70 >* ≡

```
void C_ordered_Xfactor
(
  < maxstat Xfactor Variables 69b >
) {
  < Setup maxstat Variables 71 >

  < Setup maxstat Memory 72 >

  wmax[0] = NA_INTEGER;

  for (int p = 0; p < P; p++) {
    sumleft += ExpX[p];
    sumright -= ExpX[p];

    for (int q = 0; q < Q; q++) {
      mlinstat[q] += linstat[q * P + p];
      for (R_xlen_t np = 0; np < nresample; np++)
        mblinstat[q + np * Q] += blinstat[q * P + p + np * PQ];
      mexpect[q] += expect[q * P + p];
      if (B == 1) {
        < Compute maxstat Variance / Covariance Directly 73b >
      } else {
        < Compute maxstat Variance / Covariance from Total Covariance 73a >
      }
    }

    if ((sumleft >= minbucket) && (sumright >= minbucket) && (ExpX[p] > 0)) {
      ls = mlinstat;
      /* compute MPinv only once */
      if (teststat != TESTSTAT_maximum)
        C_MPinv_sym(mcovar, Q, tol, mMPinv, &rank);
      < Compute maxstat Test Statistic 73c >
      if (tmp > maxstat[0]) {
        wmax[0] = p;
        maxstat[0] = tmp;
      }

      for (R_xlen_t np = 0; np < nresample; np++) {
        ls = mblinstat + np * Q;
        < Compute maxstat Test Statistic 73c >
        if (tmp > bmaxstat[np])
          bmaxstat[np] = tmp;
      }
    }
  }
  < Compute maxstat Permutation P-Value 73d >
  R_Free(mlinstat); R_Free(mexpect); R_Free(mblinstat);
  R_Free(mvar); R_Free(mcovar); R_Free(mMPinv);
  if (nresample == 0) R_Free(blinstat);
}
◇
```

Fragment referenced in [57a](#).

Defines: [C\\_ordered\\_Xfactor 35b](#), [44](#), [56](#).

Uses: [B 27a](#), [P 24a](#), [Q 24e](#).

*⟨ Setup maxstat Variables 71 ⟩* ≡

```
double *linstat, *expect, *covar, *varinf, *covinf, *ExpX, *blinstat, tol, *ls;
int P, Q, B;
R_xlen_t nresample;

double *mlinstat, *mblinstat, *mexpect, *mvar, *mcovar, *mMPinv,
      tmp, sumleft, sumright, sumweights;
int rank, PQ, greater;

Q = C_get_Q(LECV);
P = C_get_P(LECV);
PQ = mPQB(P, Q, 1);
B = C_get_B(LECV);
if (B > 1) {
    if (C_get_varonly(LECV))
        error("need covariance for maximally statistics with blocks");
    covar = C_get_Covariance(LECV);
} else {
    covar = C_get_Variance(LECV); /* make -Wall happy */
}
linstat = C_get_LinearStatistic(LECV);
expect = C_get_Expectation(LECV);
ExpX = C_get_ExpectationX(LECV);
/* both need to be there */
varinf = C_get_VarianceInfluence(LECV);
covinf = C_get_CovarianceInfluence(LECV);
nresample = C_get_nresample(LECV);
if (nresample > 0)
    blinstat = C_get_PermutedLinearStatistic(LECV);
tol = C_get_tol(LECV);
◇
```

Fragment referenced in [70](#), [74](#).

Uses: [B 27a](#), [C\\_get\\_B 146a](#), [C\\_get\\_Covariance 143c](#), [C\\_get\\_CovarianceInfluence 144c](#), [C\\_get\\_Expectation 143a](#),  
[C\\_get\\_ExpectationX 144a](#), [C\\_get\\_LinearStatistic 142d](#), [C\\_get\\_nresample 146b](#), [C\\_get\\_P 141c](#),  
[C\\_get\\_PermutedLinearStatistic 146c](#), [C\\_get\\_Q 142a](#), [C\\_get\\_tol 146d](#), [C\\_get\\_Variance 143b](#),  
[C\\_get\\_VarianceInfluence 144d](#), [C\\_get\\_varonly 142b](#), [LECV 141b](#), [mPQB 132a](#), [P 24a](#), [Q 24e](#), [sumweights 25e](#).

⟨*Setup maxstat Memory 72*⟩ ≡

```
mllnstat = R_Calloc(Q, double);
mexpect = R_Calloc(Q, double);
if (teststat == TESTSTAT_maximum) {
    mvar = R_Calloc(Q, double);
    /* not needed, but allocate anyway to make -Wmaybe-uninitialized happy */
    mcovar = R_Calloc(1, double);
    mMPinv = R_Calloc(1, double);
} else {
    mcovar = R_Calloc(Q * (Q + 1) / 2, double);
    mMPinv = R_Calloc(Q * (Q + 1) / 2, double);
    /* not needed, but allocate anyway to make -Wmaybe-uninitialized happy */
    mvar = R_Calloc(1, double);
}
if (nresample > 0) {
    mblnstat = R_Calloc(Q * nresample, double);
} else { /* not needed, but allocate anyway to make -Wmaybe-uninitialized happy */
    mblnstat = R_Calloc(1, double);
    blnstat = R_Calloc(1, double);
}

maxstat[0] = 0.0;

for (int q = 0; q < Q; q++) {
    mllnstat[q] = 0.0;
    mexpect[q] = 0.0;
    if (teststat == TESTSTAT_maximum)
        mvar[q] = 0.0;
    for (R_xlen_t np = 0; np < nresample; np++) {
        mblnstat[q + np * Q] = 0.0;
        bmaxstat[np] = 0.0;
    }
}
if (teststat == TESTSTAT_quadratic) {
    for (int q = 0; q < Q * (Q + 1) / 2; q++)
        mcovar[q] = 0.0;
}

sumleft = 0.0;
sumright = 0.0;
for (int p = 0; p < P; p++)
    sumright += ExpX[p];
sumweights = sumright;
◇
```

Fragment referenced in [70](#), [74](#).

Uses: [P 24a](#), [Q 24e](#), [sumweights 25e](#).

⟨ *Compute maxstat Variance / Covariance from Total Covariance 73a* ⟩ ≡

```
if (teststat == TESTSTAT_maximum) {
    for (int pp = 0; pp < p; pp++)
        mvar[q] += 2 * covar[S(pp + q * P, p + P * q, mPQB(P, Q, 1))];
    mvar[q] += covar[S(p + q * P, p + P * q, mPQB(P, Q, 1))];
} else {
    for (int qq = 0; qq <= q; qq++) {
        for (int pp = 0; pp < p; pp++)
            mcovar[S(q, qq, Q)] += 2 * covar[S(pp + q * P, p + P * qq, mPQB(P, Q, 1))];
        mcovar[S(q, qq, Q)] += covar[S(p + q * P, p + P * qq, mPQB(P, Q, 1))];
    }
}
}
◇
```

Fragment referenced in 70.

Uses: mPQB 132a, P 24a, Q 24e, S 21a.

⟨ *Compute maxstat Variance / Covariance Directly 73b* ⟩ ≡

```
/* does not work with blocks! */
if (teststat == TESTSTAT_maximum) {
    C_VarianceLinearStatistic(1, Q, varinf, &sumleft, &sumleft,
                             sumweights, 0, mvar);
} else {
    C_CovarianceLinearStatistic(1, Q, covinf, &sumleft, &sumleft,
                               sumweights, 0, mcovar);
}
}
◇
```

Fragment referenced in 70.

Uses: C\_CovarianceLinearStatistic 79, C\_VarianceLinearStatistic 80a, Q 24e, sumweights 25e.

⟨ *Compute maxstat Test Statistic 73c* ⟩ ≡

```
if (teststat == TESTSTAT_maximum) {
    tmp = C_maxtype(Q, ls, mexpect, mvar, 1, tol,
                  ALTERNATIVE_twosided);
} else {
    tmp = C_quadform(Q, ls, mexpect, mPInv);
}
}
◇
```

Fragment referenced in 70, 74.

Uses: C\_maxtype 63, C\_quadform 62, Q 24e.

⟨ *Compute maxstat Permutation P-Value 73d* ⟩ ≡

```
if (nresample > 0) {
    greater = 0;
    for (R_xlen_t np = 0; np < nresample; np++) {
        if (bmaxstat[np] > maxstat[0]) greater++;
    }
    pval[0] = C_perm_pvalue(greater, nresample, lower, give_log);
}
}
◇
```

Fragment referenced in 70, 74.

Uses: C\_perm\_pvalue 65.

*< C\_unordered\_Xfactor 74 >* ≡

```
void C_unordered_Xfactor
(
  < maxstat Xfactor Variables 69b >
) {
  double *mtmp;
  int qPp, nc, *levels, Pnonzero, *indl, *contrast;

  < Setup maxstat Variables 71 >

  < Setup maxstat Memory 72 >
  mtmp = R_Calloc(P, double);

  for (int p = 0; p < P; p++) wmax[p] = NA_INTEGER;

  < Count Levels 75a >

  for (int j = 1; j < mi; j++) { /* go though all splits */

    < Setup unordered maxstat Contrasts 75b >

    < Compute unordered maxstat Linear Statistic and Expectation 76a >

    if (B == 1) {
      < Compute unordered maxstat Variance / Covariance Directly 77a >
    } else {
      < Compute unordered maxstat Variance / Covariance from Total Covariance 76b >
    }

    if ((sumleft >= minbucket) && (sumright >= minbucket)) {
      ls = mlinstat;
      /* compute MPinv only once */
      if (teststat != TESTSTAT_maximum)
        C_MPinv_sym(mcovar, Q, tol, mMPinv, &rank);
      < Compute maxstat Test Statistic 73c >
      if (tmp > maxstat[0]) {
        for (int p = 0; p < Pnonzero; p++)
          wmax[levels[p]] = contrast[levels[p]];
        maxstat[0] = tmp;
      }

      for (R_xlen_t np = 0; np < nresample; np++) {
        ls = mblinstat + np * Q;
        < Compute maxstat Test Statistic 73c >
        if (tmp > bmaxstat[np])
          bmaxstat[np] = tmp;
      }
    }
  }

  < Compute maxstat Permutation P-Value 73d >

  R_Free(mlinstat); R_Free(mexpect); R_Free(levels); R_Free(contrast); R_Free(indl); R_Free(mtmp);
  R_Free(mblinstat); R_Free(mvar); R_Free(mcovar); R_Free(mMPinv);
  if (nresample == 0) R_Free(blinstat);
}
◇
```

Fragment referenced in [57a](#).

Defines: `C_unordered_Xfactor` [35b](#), [56](#).

Uses: `B` [27a](#), `P` [24a](#), `Q` [24e](#).

*< Count Levels 75a >* ≡

```
contrast = R_Calloc(P, int);
Pnonzero = 0;
for (int p = 0; p < P; p++) {
    if (ExpX[p] > 0) Pnonzero++;
}
levels = R_Calloc(Pnonzero, int);
nc = 0;
for (int p = 0; p < P; p++) {
    if (ExpX[p] > 0) {
        levels[nc] = p;
        nc++;
    }
}

if (Pnonzero >= 31)
    error("cannot search for unordered splits in >= 31 levels");

int mi = 1;
for (int l = 1; l < Pnonzero; l++) mi *= 2;
indl = R_Calloc(Pnonzero, int);
for (int p = 0; p < Pnonzero; p++) indl[p] = 0;
◇
```

Fragment referenced in [74](#).

Uses: P [24a](#).

*< Setup unordered maxstat Contrasts 75b >* ≡

```
/* indl determines if level p is left or right */
int jj = j;
for (int l = 1; l < Pnonzero; l++) {
    indl[l] = (jj%2);
    jj /= 2;
}

sumleft = 0.0;
sumright = 0.0;
for (int p = 0; p < P; p++) contrast[p] = 0;
for (int p = 0; p < Pnonzero; p++) {
    sumleft += indl[p] * ExpX[levels[p]];
    sumright += (1 - indl[p]) * ExpX[levels[p]];
    contrast[levels[p]] = indl[p];
}
◇
```

Fragment referenced in [74](#).

Uses: P [24a](#).

*⟨ Compute unordered maxstat Linear Statistic and Expectation 76a ⟩ ≡*

```

for (int q = 0; q < Q; q++) {
  mlinstat[q] = 0.0;
  mexpect[q] = 0.0;
  for (R_xlen_t np = 0; np < nresample; np++)
    mblinstat[q + np * Q] = 0.0;
  for (int p = 0; p < P; p++) {
    qPp = q * P + p;
    mlinstat[q] += contrast[p] * linstat[qPp];
    mexpect[q] += contrast[p] * expect[qPp];
    for (R_xlen_t np = 0; np < nresample; np++)
      mblinstat[q + np * Q] += contrast[p] * blinstat[q * P + p + np * PQ];
  }
}
◇

```

Fragment referenced in 74.

Uses: P 24a, Q 24e.

*⟨ Compute unordered maxstat Variance / Covariance from Total Covariance 76b ⟩ ≡*

```

if (teststat == TESTSTAT_maximum) {
  for (int q = 0; q < Q; q++) {
    mvar[q] = 0.0;
    for (int p = 0; p < P; p++) {
      qPp = q * P + p;
      mtmp[p] = 0.0;
      for (int pp = 0; pp < P; pp++)
        mtmp[p] += contrast[pp] * covar[S(pp + q * P, qPp, PQ)];
    }
    for (int p = 0; p < P; p++)
      mvar[q] += contrast[p] * mtmp[p];
  }
} else {
  for (int q = 0; q < Q; q++) {
    for (int qq = 0; qq <= q; qq++)
      mcovar[S(q, qq, Q)] = 0.0;
    for (int qq = 0; qq <= q; qq++) {
      for (int p = 0; p < P; p++) {
        mtmp[p] = 0.0;
        for (int pp = 0; pp < P; pp++)
          mtmp[p] += contrast[pp] * covar[S(pp + q * P, p + P * qq,
            mPQB(P, Q, 1))];
      }
      for (int p = 0; p < P; p++)
        mcovar[S(q, qq, Q)] += contrast[p] * mtmp[p];
    }
  }
}
◇

```

Fragment referenced in 74.

Uses: mPQB 132a, P 24a, Q 24e, S 21a.

*⟨ Compute unordered maxstat Variance / Covariance Directly 77a ⟩* ≡

```
if (teststat == TESTSTAT_maximum) {
    C_VarianceLinearStatistic(1, Q, varinf, &sumleft, &sumleft,
                             sumweights, 0, mvar);
} else {
    C_CovarianceLinearStatistic(1, Q, covinf, &sumleft, &sumleft,
                               sumweights, 0, mcovar);
}
◇
```

Fragment referenced in [74](#).

Uses: [C\\_CovarianceLinearStatistic 79](#), [C\\_VarianceLinearStatistic 80a](#), [Q 24e](#), [sumweights 25e](#).

### 3.7 Linear Statistics

*⟨ LinearStatistics 77b ⟩* ≡

```
⟨ RC_LinearStatistic 77d ⟩
◇
```

Fragment referenced in [23a](#).

*⟨ RC\_LinearStatistic Prototype 77c ⟩* ≡

```
void RC_LinearStatistic
(
    ⟨ R x Input 23d ⟩
    ⟨ C integer N Input 23c ⟩,
    ⟨ C integer P Input 24a ⟩,
    ⟨ C real y Input 24f ⟩
    ⟨ R weights Input 25b ⟩,
    ⟨ R subset Input 26a ⟩,
    ⟨ C subset range Input 26c ⟩,
    ⟨ C KronSums Answer 96a ⟩
)
◇
```

Fragment referenced in [77d](#).

Uses: [RC\\_LinearStatistic 77d](#).

*⟨ RC\_LinearStatistic 77d ⟩* ≡

```
⟨ RC_LinearStatistic Prototype 77c ⟩
{
    double center;

    RC_KronSums(x, N, P, y, Q, !DoSymmetric, &center, &center, !DoCenter, weights,
               subset, offset, Nsubset, PQ_ans);
}
◇
```

Fragment referenced in [77b](#).

Defines: [RC\\_LinearStatistic 33b](#), [77c](#).

Uses: [DoCenter 21b](#), [DoSymmetric 21b](#), [N 23bc](#), [Nsubset 26b](#), [offset 26c](#), [P 24a](#), [Q 24e](#), [RC\\_KronSums 95b](#), [subset 26ade](#), [weights 25b](#), [weights, 25cd](#), [x 23d, 24bc](#), [y 24df, 25a](#).

## 3.8 Expectation and Covariance

*< ExpectationCovariances 78a >* ≡

*< RC\_ExpectationInfluence 81c >*  
*< R\_ExpectationInfluence 81a >*  
*< RC\_CovarianceInfluence 84a >*  
*< R\_CovarianceInfluence 83a >*  
*< RC\_ExpectationX 85b >*  
*< R\_ExpectationX 84c >*  
*< RC\_CovarianceX 88a >*  
*< R\_CovarianceX 87a >*  
*< C\_ExpectationLinearStatistic 78b >*  
*< C\_CovarianceLinearStatistic 79 >*  
*< C\_VarianceLinearStatistic 80a >*  
◇

Fragment referenced in [23a](#).

### 3.8.1 Linear Statistic

*< C\_ExpectationLinearStatistic 78b >* ≡

```
void C_ExpectationLinearStatistic
(
    < C integer P Input 24a >,
    < C integer Q Input 24e >,
    double *ExpInf,
    double *ExpX,
    const int add,
    double *PQ_ans
) {
    if (!add)
        for (int p = 0; p < mPQB(P, Q, 1); p++) PQ_ans[p] = 0.0;

    for (int p = 0; p < P; p++) {
        for (int q = 0; q < Q; q++)
            PQ_ans[q * P + p] += ExpX[p] * ExpInf[q];
    }
}
◇
```

Fragment referenced in [78a](#).

Defines: *C\_ExpectationLinearStatistic* [35a](#), [43b](#).

Uses: *mPQB* [132a](#), *P* [24a](#), *Q* [24e](#).

*< C\_CovarianceLinearStatistic 79 >* ≡

```
void C_CovarianceLinearStatistic
(
  < C integer P Input 24a >,
  < C integer Q Input 24e >,
  double *CovInf,
  double *ExpX,
  double *CovX,
  < C sumweights Input 25e >,
  const int add,
  double *PQPQ_sym_ans
) {
  double f1 = sumweights / (sumweights - 1);
  double f2 = 1.0 / (sumweights - 1);
  double tmp, *PP_sym_tmp;

  if (mPQB(P, Q, 1) == 1) {
    tmp = f1 * CovInf[0] * CovX[0];
    tmp -= f2 * CovInf[0] * ExpX[0] * ExpX[0];
    if (add) {
      PQPQ_sym_ans[0] += tmp;
    } else {
      PQPQ_sym_ans[0] = tmp;
    }
  } else {
    PP_sym_tmp = R_Calloc(PP12(P), double);
    C_KronSums_sym_(ExpX, 1, P,
                   PP_sym_tmp);
    for (int p = 0; p < PP12(P); p++)
      PP_sym_tmp[p] = f1 * CovX[p] - f2 * PP_sym_tmp[p];
    C_kronecker_sym(CovInf, Q, PP_sym_tmp, P, 1 - (add >= 1),
                   PQPQ_sym_ans);
    R_Free(PP_sym_tmp);
  }
}
◇
```

Fragment referenced in [78a](#).

Defines: [C\\_CovarianceLinearStatistic 35d](#), [44](#), [73b](#), [77a](#), [80a](#).

Uses: [C\\_kronecker\\_sym 134](#), [mPQB 132a](#), [P 24a](#), [PP12 131b](#), [Q 24e](#), [sumweights 25e](#).

*< C\_VarianceLinearStatistic 80a >* ≡

```
void C_VarianceLinearStatistic
(
  < C integer P Input 24a >,
  < C integer Q Input 24e >,
  double *VarInf,
  double *ExpX,
  double *VarX,
  < C sumweights Input 25e >,
  const int add,
  double *PQ_ans
) {
  if (mPQB(P, Q, 1) == 1) {
    C_CovarianceLinearStatistic(P, Q, VarInf, ExpX, VarX,
                               sumweights, (add >= 1),
                               PQ_ans);
  } else {
    double *P_tmp;
    P_tmp = R_Calloc(P, double);
    double f1 = sumweights / (sumweights - 1);
    double f2 = 1.0 / (sumweights - 1);
    for (int p = 0; p < P; p++)
      P_tmp[p] = f1 * VarX[p] - f2 * ExpX[p] * ExpX[p];
    C_kronecker(VarInf, 1, Q, P_tmp, 1, P, 1 - (add >= 1),
               PQ_ans);
    R_Free(P_tmp);
  }
}
◇
```

Fragment referenced in [78a](#).

Defines: [C\\_VarianceLinearStatistic 35c, 44, 73b, 77a](#).

Uses: [C\\_CovarianceLinearStatistic 79](#), [C\\_kronecker 133b](#), [mPQB 132a](#), [P 24a](#), [Q 24e](#), [sumweights 25e](#).

### 3.8.2 Influence

```
> sumweights <- sum(weights[subset])
> expecty <- colSums(y[subset, ] * weights[subset]) / sumweights
> a0 <- expecty
> a1 <- .Call(libcoin:::R_ExpectationInfluence, y, weights, subset)
> a2 <- .Call(libcoin:::R_ExpectationInfluence, y, as.double(weights), as.double(subset))
> a3 <- .Call(libcoin:::R_ExpectationInfluence, y, weights, as.double(subset))
> a4 <- .Call(libcoin:::R_ExpectationInfluence, y, as.double(weights), subset)
> a5 <- LinStatExpCov(x, y, weights = weights, subset = subset)$ExpectationInfluence
> stopifnot(isequal(a0, a1) && isequal(a0, a2) &&
+           isequal(a0, a3) && isequal(a0, a4) &&
+           isequal(a0, a5))
```

*< R\_ExpectationInfluence Prototype 80b >* ≡

```
SEXP R_ExpectationInfluence
(
  < R y Input 24d >
  < R weights Input 25b >,
  < R subset Input 26a >
)
◇
```

Fragment referenced in [22b, 81a](#).

Uses: [R\\_ExpectationInfluence 81a](#).

$\langle R\_ExpectationInfluence\ 81a \rangle \equiv$

```
 $\langle R\_ExpectationInfluence\ Prototype\ 80b \rangle$ 
{
    SEXP ans;
     $\langle C\ integer\ Q\ Input\ 24e \rangle$ ;
     $\langle C\ integer\ N\ Input\ 23c \rangle$ ;
     $\langle C\ integer\ Nsubset\ Input\ 26b \rangle$ ;
    double sumweights;

    Q = NCOL(y);
    N = XLENGTH(y) / Q;
    Nsubset = XLENGTH(subset);

    sumweights = RC_Sums(N, weights, subset, Offset0, Nsubset);

    PROTECT(ans = allocVector(REALSXP, Q));
    RC_ExpectationInfluence(N, y, Q, weights, subset, Offset0, Nsubset, sumweights, REAL(ans));
    UNPROTECT(1);
    return(ans);
}
◇
```

Fragment referenced in 78a.

Defines: R\_ExpectationInfluence 80b, 83a, 152c, 153.

Uses: N 23bc, NCOL 130c, Nsubset 26b, Offset0 21b, Q 24e, RC\_ExpectationInfluence 81c, RC\_Sums 91a, subset 26ade, sumweights 25e, weights 25b, weights, 25cd, y 24df, 25a.

$\langle RC\_ExpectationInfluence\ Prototype\ 81b \rangle \equiv$

```
void RC_ExpectationInfluence
(
     $\langle C\ integer\ N\ Input\ 23c \rangle$ ,
     $\langle R\ y\ Input\ 24d \rangle$ 
     $\langle C\ integer\ Q\ Input\ 24e \rangle$ ,
     $\langle R\ weights\ Input\ 25b \rangle$ ,
     $\langle R\ subset\ Input\ 26a \rangle$ ,
     $\langle C\ subset\ range\ Input\ 26c \rangle$ ,
     $\langle C\ sumweights\ Input\ 25e \rangle$ ,
     $\langle C\ colSums\ Answer\ 108a \rangle$ 
)
◇
```

Fragment referenced in 81c.

Uses: RC\_ExpectationInfluence 81c.

$\langle RC\_ExpectationInfluence\ 81c \rangle \equiv$

```
 $\langle RC\_ExpectationInfluence\ Prototype\ 81b \rangle$ 
{
    double center;

    RC_colSums(REAL(y), N, Q, Power1, &center, !DoCenter, weights,
              subset, offset, Nsubset, P_ans);
    for (int q = 0; q < Q; q++)
        P_ans[q] = P_ans[q] / sumweights;
}
◇
```

Fragment referenced in 78a.

Defines: RC\_ExpectationInfluence 35a, 43b, 81ab.

Uses: DoCenter 21b, N 23bc, Nsubset 26b, offset 26c, Power1 21b, Q 24e, RC\_colSums 107b, subset 26ade, sumweights 25e, weights 25b, weights, 25cd, y 24df, 25a.

```

> sumweights <- sum(weights[subset])
> yc <- t(t(y) - expecty)
> r1y <- rep(1:ncol(y), ncol(y))
> r2y <- rep(1:ncol(y), each = ncol(y))
> a0 <- colSums(yc[subset, r1y] * yc[subset, r2y] * weights[subset]) / sumweights
> a0 <- matrix(a0, ncol = ncol(y))
> vary <- diag(a0)
> a0 <- a0[lower.tri(a0, diag = TRUE)]
> a1 <- .Call(libcoin:::R_CovarianceInfluence, y, weights, subset, 0L)
> a2 <- .Call(libcoin:::R_CovarianceInfluence, y, as.double(weights), as.double(subset), 0L)
> a3 <- .Call(libcoin:::R_CovarianceInfluence, y, weights, as.double(subset), 0L)
> a4 <- .Call(libcoin:::R_CovarianceInfluence, y, as.double(weights), subset, 0L)
> a5 <- LinStatExpCov(x, y, weights = weights, subset = subset)$CovarianceInfluence
> stopifnot(isequal(a0, a1) && isequal(a0, a2) &&
+           isequal(a0, a3) && isequal(a0, a4) &&
+           isequal(a0, a5))
> a0 <- vary
> a1 <- .Call(libcoin:::R_CovarianceInfluence, y, weights, subset, 1L)
> a2 <- .Call(libcoin:::R_CovarianceInfluence, y, as.double(weights), as.double(subset), 1L)
> a3 <- .Call(libcoin:::R_CovarianceInfluence, y, weights, as.double(subset), 1L)
> a4 <- .Call(libcoin:::R_CovarianceInfluence, y, as.double(weights), subset, 1L)
> a5 <- LinStatExpCov(x, y, weights = weights, subset = subset, varonly = TRUE)$VarianceInfluence
> stopifnot(isequal(a0, a1) && isequal(a0, a2) &&
+           isequal(a0, a3) && isequal(a0, a4) &&
+           isequal(a0, a5))

```

*<R\_CovarianceInfluence Prototype 82> ≡*

```

SEXP R_CovarianceInfluence
(
  <R y Input 24d>
  <R weights Input 25b>,
  <R subset Input 26a>,
  SEXP varonly
)
◇

```

Fragment referenced in [22b](#), [83a](#).

Uses: [R\\_CovarianceInfluence 83a](#).

$\langle R\_CovarianceInfluence\ 83a \rangle \equiv$

```
 $\langle R\_CovarianceInfluence\ Prototype\ 82 \rangle$ 
{
  SEXP ans;
  SEXP ExpInf;
   $\langle C\ integer\ Q\ Input\ 24e \rangle$ ;
   $\langle C\ integer\ N\ Input\ 23c \rangle$ ;
   $\langle C\ integer\ Nsubset\ Input\ 26b \rangle$ ;
  double sumweights;

  Q = NCOL(y);
  N = XLENGTH(y) / Q;
  Nsubset = XLENGTH(subset);

  PROTECT(ExpInf = R_ExpectationInfluence(y, weights, subset));

  sumweights = RC_Sums(N, weights, subset, Offset0, Nsubset);

  if (INTEGER(varonly)[0]) {
    PROTECT(ans = allocVector(REALSXP, Q));
  } else {
    PROTECT(ans = allocVector(REALSXP, Q * (Q + 1) / 2));
  }
  RC_CovarianceInfluence(N, y, Q, weights, subset, Offset0, Nsubset, REAL(ExpInf), sumweights,
                        INTEGER(varonly)[0], REAL(ans));
  UNPROTECT(2);
  return(ans);
}
◇
```

Fragment referenced in 78a.

Defines: `R_CovarianceInfluence` 82, 152c, 153.

Uses: `N` 23bc, `NCOL` 130c, `Nsubset` 26b, `Offset0` 21b, `Q` 24e, `RC_CovarianceInfluence` 84a, `RC_Sums` 91a, `R_ExpectationInfluence` 81a, `subset` 26ade, `sumweights` 25e, `weights` 25b, `weights`, 25cd, `y` 24df, 25a.

$\langle RC\_CovarianceInfluence\ Prototype\ 83b \rangle \equiv$

```
void RC_CovarianceInfluence
(
   $\langle C\ integer\ N\ Input\ 23c \rangle$ ,
   $\langle R\ y\ Input\ 24d \rangle$ 
   $\langle C\ integer\ Q\ Input\ 24e \rangle$ ,
   $\langle R\ weights\ Input\ 25b \rangle$ ,
   $\langle R\ subset\ Input\ 26a \rangle$ ,
   $\langle C\ subset\ range\ Input\ 26c \rangle$ ,
  double *ExpInf,
   $\langle C\ sumweights\ Input\ 25e \rangle$ ,
  int VARONLY,
   $\langle C\ KronSums\ Answer\ 96a \rangle$ 
)
◇
```

Fragment referenced in 84a.

Uses: `RC_CovarianceInfluence` 84a.

$\langle RC\_CovarianceInfluence\ 84a \rangle \equiv$

```
 $\langle RC\_CovarianceInfluence\ Prototype\ 83b \rangle$ 
{
  if (VARONLY) {
    RC_colSums(REAL(y), N, Q, Power2, ExpInf, DoCenter, weights,
              subset, offset, Nsubset, PQ_ans);
    for (int q = 0; q < Q; q++)
      PQ_ans[q] = PQ_ans[q] / sumweights;
  } else {
    RC_KronSums(y, N, Q, REAL(y), Q, DoSymmetric, ExpInf, ExpInf, DoCenter, weights,
              subset, offset, Nsubset, PQ_ans);
    for (int q = 0; q < Q * (Q + 1) / 2; q++)
      PQ_ans[q] = PQ_ans[q] / sumweights;
  }
}
◇
```

Fragment referenced in 78a.

Defines: `RC_CovarianceInfluence` 35b, 44, 83ab.

Uses: `DoCenter` 21b, `DoSymmetric` 21b, `N` 23bc, `Nsubset` 26b, `offset` 26c, `Power2` 21b, `Q` 24e, `RC_colSums` 107b, `RC_KronSums` 95b, `subset` 26ade, `sumweights` 25e, `weights` 25b, `weights`, 25cd, `y` 24df, 25a.

### 3.8.3 X

$\langle R\_ExpectationX\ Prototype\ 84b \rangle \equiv$

```
SEXP R_ExpectationX
(
   $\langle R\ x\ Input\ 23d \rangle$ 
  SEXP P,
   $\langle R\ weights\ Input\ 25b \rangle$ ,
   $\langle R\ subset\ Input\ 26a \rangle$ 
)
◇
```

Fragment referenced in 22b, 84c.

Uses: `P` 24a, `R_ExpectationX` 84c.

$\langle R\_ExpectationX\ 84c \rangle \equiv$

```
 $\langle R\_ExpectationX\ Prototype\ 84b \rangle$ 
{
  SEXP ans;
   $\langle C\ integer\ N\ Input\ 23c \rangle$ ;
   $\langle C\ integer\ Nsubset\ Input\ 26b \rangle$ ;

  N = XLENGTH(x) / INTEGER(P)[0];
  Nsubset = XLENGTH(subset);

  PROTECT(ans = allocVector(REALSXP, INTEGER(P)[0]));
  RC_ExpectationX(x, N, INTEGER(P)[0], weights, subset,
                 Offset0, Nsubset, REAL(ans));
  UNPROTECT(1);
  return(ans);
}
◇
```

Fragment referenced in 78a.

Defines: `R_ExpectationX` 84b, 87a, 152c, 153.

Uses: `N` 23bc, `Nsubset` 26b, `Offset0` 21b, `P` 24a, `RC_ExpectationX` 85b, `subset` 26ade, `weights` 25b, `weights`, 25cd, `x` 23d, 24bc.

*< RC\_ExpectationX Prototype 85a >* ≡

```
void RC_ExpectationX
(
  < R x Input 23d >
  < C integer N Input 23c >,
  < C integer P Input 24a >,
  < R weights Input 25b >,
  < R subset Input 26a >,
  < C subset range Input 26c >,
  < C OneTableSums Answer 112c >
)
◇
```

Fragment referenced in 85b.  
Uses: RC\_ExpectationX 85b.

*< RC\_ExpectationX 85b >* ≡

```
< RC_ExpectationX Prototype 85a >
{
  double center;

  if (TYPEOF(x) == INTSXP) {
    double* Pp1tmp = R_Calloc(P + 1, double);
    RC_OneTableSums(INTEGER(x), N, P + 1, weights, subset, offset, Nsubset, Pp1tmp);
    for (int p = 0; p < P; p++) P_ans[p] = Pp1tmp[p + 1];
    R_Free(Pp1tmp);
  } else {
    RC_colSums(REAL(x), N, P, Power1, &center, !DoCenter, weights, subset, offset, Nsubset, P_ans);
  }
}
◇
```

Fragment referenced in 78a.

Defines: RC\_ExpectationX 35a, 43b, 84c, 85a.

Uses: DoCenter 21b, N 23bc, Nsubset 26b, offset 26c, P 24a, Power1 21b, RC\_colSums 107b, RC\_OneTableSums 112a, subset 26ade, weights 25b, weights, 25cd, x 23d, 24bc.

```
> a0 <- colSums(x[subset, ] * weights[subset])
> a1 <- .Call(libcoin:::R_ExpectationX, x, P, weights, subset);
> a2 <- .Call(libcoin:::R_ExpectationX, x, P, as.double(weights), as.double(subset))
> a3 <- .Call(libcoin:::R_ExpectationX, x, P, weights, as.double(subset))
> a4 <- .Call(libcoin:::R_ExpectationX, x, P, as.double(weights), subset)
> stopifnot(isequal(a0, a1) && isequal(a0, a2) &&
+           isequal(a0, a3) && isequal(a0, a4) &&
+           isequal(a0, LECVxyws$ExpectationX))
> a0 <- colSums(x[subset, ]^2 * weights[subset])
> a1 <- .Call(libcoin:::R_CovarianceX, x, P, weights, subset, 1L)
> a2 <- .Call(libcoin:::R_CovarianceX, x, P, as.double(weights), as.double(subset), 1L)
> a3 <- .Call(libcoin:::R_CovarianceX, x, P, weights, as.double(subset), 1L)
> a4 <- .Call(libcoin:::R_CovarianceX, x, P, as.double(weights), subset, 1L)
> stopifnot(isequal(a0, a1) && isequal(a0, a2) &&
+           isequal(a0, a3) && isequal(a0, a4))
> a0 <- as.vector(colSums(Xfactor[subset, ] * weights[subset]))
> a1 <- .Call(libcoin:::R_ExpectationX, ix, Lx, weights, subset)
> a2 <- .Call(libcoin:::R_ExpectationX, ix, Lx, as.double(weights), as.double(subset))
> a3 <- .Call(libcoin:::R_ExpectationX, ix, Lx, weights, as.double(subset))
> a4 <- .Call(libcoin:::R_ExpectationX, ix, Lx, as.double(weights), subset)
> stopifnot(isequal(a0, a1) && isequal(a0, a2) &&
+           isequal(a0, a3) && isequal(a0, a4))
```

```

> a1 <- .Call(libcoin:::R_CovarianceX, ix, Lx, weights, subset, 1L)
> a2 <- .Call(libcoin:::R_CovarianceX, ix, Lx, as.double(weights), as.double(subset), 1L)
> a3 <- .Call(libcoin:::R_CovarianceX, ix, Lx, weights, as.double(subset), 1L)
> a4 <- .Call(libcoin:::R_CovarianceX, ix, Lx, as.double(weights), subset, 1L)
> stopifnot(isequal(a0, a1) && isequal(a0, a2) &&
+           isequal(a0, a3) && isequal(a0, a4))
> r1x <- rep(1:ncol(Xfactor), ncol(Xfactor))
> r2x <- rep(1:ncol(Xfactor), each = ncol(Xfactor))
> a0 <- colSums(Xfactor[subset, r1x] * Xfactor[subset, r2x] * weights[subset])
> a0 <- matrix(a0, ncol = ncol(Xfactor))
> vary <- diag(a0)
> a0 <- a0[lower.tri(a0, diag = TRUE)]
> a1 <- .Call(libcoin:::R_CovarianceX, ix, Lx, weights, subset, 0L)
> a2 <- .Call(libcoin:::R_CovarianceX, ix, Lx, as.double(weights), as.double(subset), 0L)
> a3 <- .Call(libcoin:::R_CovarianceX, ix, Lx, weights, as.double(subset), 0L)
> a4 <- .Call(libcoin:::R_CovarianceX, ix, Lx, as.double(weights), subset, 0L)
> stopifnot(isequal(a0, a1) && isequal(a0, a2) &&
+           isequal(a0, a3) && isequal(a0, a4))

```

*<R\_CovarianceX Prototype 86>* ≡

```

SEXP R_CovarianceX
(
  <R x Input 23d>
  SEXP P,
  <R weights Input 25b>,
  <R subset Input 26a>,
  SEXP varonly
)
◇

```

Fragment referenced in [22b](#), [87a](#).

Uses: P [24a](#), R\_CovarianceX [87a](#).

$\langle R\_CovarianceX\ 87a \rangle \equiv$

```
 $\langle R\_CovarianceX\ Prototype\ 86 \rangle$ 
{
  SEXP ans;
  SEXP ExpX;
   $\langle C\ integer\ N\ Input\ 23c \rangle$ ;
   $\langle C\ integer\ Nsubset\ Input\ 26b \rangle$ ;

  N = XLENGTH(x) / INTEGER(P)[0];
  Nsubset = XLENGTH(subset);

  PROTECT(ExpX = R_ExpectationX(x, P, weights, subset));

  if (INTEGER(varonly)[0]) {
    PROTECT(ans = allocVector(REALSXP, INTEGER(P)[0]));
  } else {
    PROTECT(ans = allocVector(REALSXP, INTEGER(P)[0] * (INTEGER(P)[0] + 1) / 2));
  }
  RC_CovarianceX(x, N, INTEGER(P)[0], weights, subset, Offset0, Nsubset, REAL(ExpX),
    INTEGER(varonly)[0], REAL(ans));
  UNPROTECT(2);
  return(ans);
}
◇
```

Fragment referenced in 78a.

Defines: R\_CovarianceX 86, 152c, 153.

Uses: N 23bc, Nsubset 26b, Offset0 21b, P 24a, RC\_CovarianceX 88a, R\_ExpectationX 84c, subset 26ade, weights 25b, weights, 25cd, x 23d, 24bc.

$\langle RC\_CovarianceX\ Prototype\ 87b \rangle \equiv$

```
void RC_CovarianceX
(
   $\langle R\ x\ Input\ 23d \rangle$ 
   $\langle C\ integer\ N\ Input\ 23c \rangle$ ,
   $\langle C\ integer\ P\ Input\ 24a \rangle$ ,
   $\langle R\ weights\ Input\ 25b \rangle$ ,
   $\langle R\ subset\ Input\ 26a \rangle$ ,
   $\langle C\ subset\ range\ Input\ 26c \rangle$ ,
  double *ExpX,
  int VARONLY,
   $\langle C\ KronSums\ Answer\ 96a \rangle$ 
)
◇
```

Fragment referenced in 88a.

Uses: RC\_CovarianceX 88a.

*< RC\_CovarianceX 88a >* ≡

```
< RC_CovarianceX Prototype 87b >
{
    double center;

    if (TYPEOF(x) == INTSXP) {
        if (VARONLY) {
            for (int p = 0; p < P; p++) PQ_ans[p] = ExpX[p];
        } else {
            for (int p = 0; p < PP12(P); p++)
                PQ_ans[p] = 0.0;
            for (int p = 0; p < P; p++)
                PQ_ans[S(p, p, P)] = ExpX[p];
        }
    } else {
        if (VARONLY) {
            RC_colSums(REAL(x), N, P, Power2, &center, !DoCenter, weights,
                subset, offset, Nsubset, PQ_ans);
        } else {
            RC_KronSums(x, N, P, REAL(x), P, DoSymmetric, &center, &center, !DoCenter, weights,
                subset, offset, Nsubset, PQ_ans);
        }
    }
}
}
◇
```

Fragment referenced in [78a](#).

Defines: [RC\\_CovarianceX 35cd, 44, 87ab](#).

Uses: [DoCenter 21b](#), [DoSymmetric 21b](#), [N 23bc](#), [Nsubset 26b](#), [offset 26c](#), [P 24a](#), [Power2 21b](#), [PP12 131b](#), [RC\\_colSums 107b](#),  
[RC\\_KronSums 95b](#), [S 21a](#), [subset 26ade](#), [weights 25b](#), [weights, 25cd](#), [x 23d, 24bc](#).

### 3.9 Computing Sums

The core concept of all functions in the section is the computation of various sums over observations, case weights, or blocks. We start with an initialisation of the loop over all observations

*< init subset loop 88b >* ≡

```
R_xlen_t diff = 0;
s = subset + offset;
w = weights;
/* subset is R-style index in 1:N */
if (Nsubset > 0)
    diff = (R_xlen_t) s[0] - 1;
◇
```

Fragment referenced in [93a, 99, 101b, 109b, 114a, 118b, 122b](#).

Uses: [N 23bc](#), [Nsubset 26b](#), [offset 26c](#), [subset 26ade](#), [weights 25b](#).

and loop over  $i = 1, \dots, N$  when no subset was specified or over the subset of the subset given by `offset` and `Nsubset`, allowing for number of observations larger than `INT_MAX`

*< start subset loop 88c >* ≡

```
for (R_xlen_t i = 0; i < (Nsubset == 0 ? N : Nsubset) - 1; i++)
◇
```

Fragment referenced in [93a, 99, 101b, 109b, 114a, 118b, 122b](#).

Uses: [N 23bc](#), [Nsubset 26b](#).

After computations in the loop, we compute the next element

*< continue subset loop 89a >* ≡

```
if (Nsubset > 0) {
  /* NB: diff also works with R style index */
  diff = (R_xlen_t) s[1] - s[0];
  if (diff < 0)
    error("subset not sorted");
  s++;
} else {
  diff = 1;
}
◇
```

Fragment referenced in [93a](#), [99](#), [101b](#), [109b](#), [114a](#), [118b](#), [122b](#).  
Uses: [Nsubset 26b](#), [subset 26ade](#).

### 3.9.1 Simple Sums

*< SimpleSums 89b >* ≡

```
< C_Sums_dweights_dsubset 91b >
< C_Sums_iweights_dsubset 92a >
< C_Sums_iweights_isset 92b >
< C_Sums_dweights_isset 92c >
< RC_Sums 91a >
< R_Sums 90a >
◇
```

Fragment referenced in [23a](#).

```
> a0 <- sum(weights[subset])
> a1 <- .Call(libcoin:::R_Sums, N, weights, subset)
> a2 <- .Call(libcoin:::R_Sums, N, as.double(weights), as.double(subset))
> a3 <- .Call(libcoin:::R_Sums, N, weights, as.double(subset))
> a4 <- .Call(libcoin:::R_Sums, N, as.double(weights), subset)
> stopifnot(isequal(a0, a1) && isequal(a0, a2) &&
+           isequal(a0, a3) && isequal(a0, a4))
```

*< R\_Sums Prototype 89c >* ≡

```
SEXP R_Sums
(
  < R N Input 23b >
  < R weights Input 25b >,
  < R subset Input 26a >
)
◇
```

Fragment referenced in [22b](#), [90a](#).  
Uses: [R\\_Sums 90a](#).

$\langle R\_Sums\ 90a \rangle \equiv$

```
 $\langle R\_Sums\ Prototype\ 89c \rangle$ 
{
  SEXP ans;
   $\langle C\ integer\ Nsubset\ Input\ 26b \rangle$ ;

  Nsubset = XLENGTH(subset);

  PROTECT(ans = allocVector(REALSXP, 1));
  REAL(ans)[0] = RC_Sums(INTEGER(N)[0], weights, subset, Offset0, Nsubset);
  UNPROTECT(1);

  return(ans);
}
◇
```

Fragment referenced in [89b](#).

Defines: [R\\_Sums 89c](#), [152c](#), [153](#).

Uses: [N 23bc](#), [Nsubset 26b](#), [Offset0 21b](#), [RC\\_Sums 91a](#), [subset 26ade](#), [weights 25b](#), [weights, 25cd](#).

$\langle RC\_Sums\ Prototype\ 90b \rangle \equiv$

```
double RC_Sums
(
   $\langle C\ integer\ N\ Input\ 23c \rangle$ ,
   $\langle R\ weights\ Input\ 25b \rangle$ ,
   $\langle R\ subset\ Input\ 26a \rangle$ ,
   $\langle C\ subset\ range\ Input\ 26c \rangle$ 
)
◇
```

Fragment referenced in [91a](#).

Uses: [RC\\_Sums 91a](#).

$\langle RC\_Sums\ 91a \rangle \equiv$

```
 $\langle RC\_Sums\ Prototype\ 90b \rangle$ 
{
  if (XLENGTH(weights) == 0) {
    if (XLENGTH(subset) == 0) {
      return((double) N);
    } else {
      return((double) Nsubset);
    }
  }
  if (TYPEOF(weights) == INTSXP) {
    if (TYPEOF(subset) == INTSXP) {
      return(C_Sums_iweights_isubset(N, INTEGER(weights), XLENGTH(weights),
                                     INTEGER(subset), offset, Nsubset));
    } else {
      return(C_Sums_iweights_dsubset(N, INTEGER(weights), XLENGTH(weights),
                                     REAL(subset), offset, Nsubset));
    }
  } else {
    if (TYPEOF(subset) == INTSXP) {
      return(C_Sums_dweights_isubset(N, REAL(weights), XLENGTH(weights),
                                     INTEGER(subset), offset, Nsubset));
    } else {
      return(C_Sums_dweights_dsubset(N, REAL(weights), XLENGTH(weights),
                                     REAL(subset), offset, Nsubset));
    }
  }
}
◇
```

Fragment referenced in [89b](#).

Defines: [RC\\_Sums 34ab](#), [81a](#), [83a](#), [90ab](#), [123c](#), [127a](#).

Uses: [C\\_Sums\\_dweights\\_dsubset 91b](#), [C\\_Sums\\_dweights\\_isubset 92c](#), [C\\_Sums\\_iweights\\_dsubset 92a](#),  
[C\\_Sums\\_iweights\\_isubset 92b](#), [N 23bc](#), [Nsubset 26b](#), [offset 26c](#), [subset 26ade](#), [weights 25b](#).

$\langle C\_Sums\_dweights\_dsubset\ 91b \rangle \equiv$

```
double C_Sums_dweights_dsubset
(
   $\langle C\ integer\ N\ Input\ 23c \rangle$ ,
   $\langle C\ real\ weights\ Input\ 25d \rangle$ 
   $\langle C\ real\ subset\ Input\ 26e \rangle$ 
) {
  double *s, *w;
   $\langle Sums\ Body\ 93a \rangle$ 
}
◇
```

Fragment referenced in [89b](#).

Defines: [C\\_Sums\\_dweights\\_dsubset 91a](#).

$\langle C\_Sums\_iweights\_dsubset\ 92a \rangle \equiv$

```
double C_Sums_iweights_dsubset
(
   $\langle C\ integer\ N\ Input\ 23c \rangle$ ,
   $\langle C\ integer\ weights\ Input\ 25c \rangle$ 
   $\langle C\ real\ subset\ Input\ 26e \rangle$ 
) {
  double *s;
  int *w;
   $\langle Sums\ Body\ 93a \rangle$ 
}
◇
```

Fragment referenced in [89b](#).

Defines: `C_Sums_iweights_dsubset` [91a](#).

$\langle C\_Sums\_iweights\_isubset\ 92b \rangle \equiv$

```
double C_Sums_iweights_isubset
(
   $\langle C\ integer\ N\ Input\ 23c \rangle$ ,
   $\langle C\ integer\ weights\ Input\ 25c \rangle$ 
   $\langle C\ integer\ subset\ Input\ 26d \rangle$ 
) {
  int *s, *w;
   $\langle Sums\ Body\ 93a \rangle$ 
}
◇
```

Fragment referenced in [89b](#).

Defines: `C_Sums_iweights_isubset` [91a](#).

$\langle C\_Sums\_dweights\_isubset\ 92c \rangle \equiv$

```
double C_Sums_dweights_isubset
(
   $\langle C\ integer\ N\ Input\ 23c \rangle$ ,
   $\langle C\ real\ weights\ Input\ 25d \rangle$ 
   $\langle C\ integer\ subset\ Input\ 26d \rangle$ 
) {
  int *s;
  double *w;
   $\langle Sums\ Body\ 93a \rangle$ 
}
◇
```

Fragment referenced in [89b](#).

Defines: `C_Sums_dweights_isubset` [91a](#).

*<Sums Body 93a>* ≡

```
double ans = 0.0;

if (Nsubset > 0) {
  if (!HAS_WEIGHTS) return((double) Nsubset);
} else {
  if (!HAS_WEIGHTS) return((double) N);
}

<init subset loop 88b>
<start subset loop 88c>
{
  w = w + diff;
  ans += w[0];
  <continue subset loop 89a>
}

w = w + diff;
ans += w[0];

return(ans);
◇
```

Fragment referenced in [91b](#), [92abc](#).

Uses: HAS\_WEIGHTS [25cd](#), N [23bc](#), Nsubset [26b](#).

### 3.9.2 Kronecker Sums

*<KronSums 93b>* ≡

```
<C_KronSums_dweights_dsubset 97b>
<C_KronSums_iweights_dsubset 97c>
<C_KronSums_iweights_isubset 98a>
<C_KronSums_dweights_isubset 98b>
<C_XfactorKronSums_dweights_dsubset 100b>
<C_XfactorKronSums_iweights_dsubset 100c>
<C_XfactorKronSums_iweights_isubset 100d>
<C_XfactorKronSums_dweights_isubset 101a>
<RC_KronSums 95b>
<R_KronSums 94b>
<C_KronSums_Permutation_isubset 104b>
<C_KronSums_Permutation_dsubset 104a>
<C_XfactorKronSums_Permutation_isubset 105b>
<C_XfactorKronSums_Permutation_dsubset 105a>
<RC_KronSums_Permutation 103b>
<R_KronSums_Permutation 102b>
◇
```

Fragment referenced in [23a](#).

```
> r1 <- rep(1:ncol(x), ncol(y))
> r2 <- rep(1:ncol(y), each = ncol(x))
> a0 <- colSums(x[subset, r1] * y[subset, r2] * weights[subset])
> a1 <- .Call(libcoin:::R_KronSums, x, P, y, weights, subset, 0L)
> a2 <- .Call(libcoin:::R_KronSums, x, P, y, as.double(weights), as.double(subset), 0L)
> a3 <- .Call(libcoin:::R_KronSums, x, P, y, weights, as.double(subset), 0L)
> a4 <- .Call(libcoin:::R_KronSums, x, P, y, as.double(weights), subset, 0L)
> stopifnot(isequal(a0, a1) && isequal(a0, a2) &&
+           isequal(a0, a3) && isequal(a0, a4))
> a0 <- as.vector(colSums(Xfactor[subset, r1Xfactor] *
+                       y[subset, r2Xfactor] * weights[subset]))
```

```

> a1 <- .Call(libcoin:::R_KronSums, ix, Lx, y, weights, subset, 0L)
> a2 <- .Call(libcoin:::R_KronSums, ix, Lx, y, as.double(weights), as.double(subset), 0L)
> a3 <- .Call(libcoin:::R_KronSums, ix, Lx, y, weights, as.double(subset), 0L)
> a4 <- .Call(libcoin:::R_KronSums, ix, Lx, y, as.double(weights), subset, 0L)
> stopifnot(isequal(a0, a1) && isequal(a0, a2) &&
+           isequal(a0, a3) && isequal(a0, a4))

```

$\langle R\_KronSums \text{ Prototype } 94a \rangle \equiv$

```

SEXP R_KronSums
(
   $\langle R \text{ } x \text{ Input } 23d \rangle$ 
  SEXP P,
   $\langle R \text{ } y \text{ Input } 24d \rangle$ 
   $\langle R \text{ } weights \text{ Input } 25b \rangle$ ,
   $\langle R \text{ } subset \text{ Input } 26a \rangle$ ,
  SEXP symmetric
)

```

Fragment referenced in [22b](#), [94b](#).  
 Uses: P [24a](#), R\_KronSums [94b](#).

$\langle R\_KronSums \text{ } 94b \rangle \equiv$

```

 $\langle R\_KronSums \text{ Prototype } 94a \rangle$ 
{
  SEXP ans;
   $\langle C \text{ } integer \text{ } Q \text{ Input } 24e \rangle$ ;
   $\langle C \text{ } integer \text{ } N \text{ Input } 23c \rangle$ ;
   $\langle C \text{ } integer \text{ } Nsubset \text{ Input } 26b \rangle$ ;

  double center;

  Q = NCOL(y);
  N = XLENGTH(y) / Q;
  Nsubset = XLENGTH(subset);

  if (INTEGER(symmetric)[0]) {
    PROTECT(ans = allocVector(REALSXP, INTEGER(P)[0] * (INTEGER(P)[0] + 1) / 2));
  } else {
    PROTECT(ans = allocVector(REALSXP, INTEGER(P)[0] * Q));
  }
  RC_KronSums(x, N, INTEGER(P)[0], REAL(y), Q, INTEGER(symmetric)[0], &center, &center,
             !DoCenter, weights, subset, Offset0, Nsubset, REAL(ans));
  UNPROTECT(1);
  return(ans);
}

```

Fragment referenced in [93b](#).  
 Defines: R\_KronSums [94a](#), [152c](#), [153](#).  
 Uses: DoCenter [21b](#), N [23bc](#), NCOL [130c](#), Nsubset [26b](#), Offset0 [21b](#), P [24a](#), Q [24e](#), RC\_KronSums [95b](#), subset [26ade](#),  
 weights [25b](#), weights, [25cd](#), x [23d](#), [24bc](#), y [24df](#), [25a](#).

*< RC\_KronSums Prototype 95a >* ≡

```
void RC_KronSums
(
    < RC KronSums Input 95c >
    < R weights Input 25b >,
    < R subset Input 26a >,
    < C subset range Input 26c >,
    < C KronSums Answer 96a >
)
◇
```

Fragment referenced in [95b](#).

Uses: [RC\\_KronSums 95b](#).

*< RC\_KronSums 95b >* ≡

```
< RC_KronSums Prototype 95a >
{
    if (typeof(x) == INTSXP) {
        < KronSums Integer x 96b >
    } else {
        < KronSums Double x 97a >
    }
}
◇
```

Fragment referenced in [93b](#).

Defines: [RC\\_KronSums 77d](#), [84a](#), [88a](#), [94b](#), [95a](#).

Uses: [x 23d](#), [24bc](#).

*< RC KronSums Input 95c >* ≡

```
< R x Input 23d >
< C integer N Input 23c >,
< C integer P Input 24a >,
< C real y Input 24f >
const int SYMMETRIC,
double *centerx,
double *centery,
const int CENTER,
◇
```

Fragment referenced in [95a](#).

*< C KronSums Input 95d >* ≡

```
< C real x Input 24b >
< C real y Input 24f >
const int SYMMETRIC,
double *centerx,
double *centery,
const int CENTER,
◇
```

Fragment referenced in [97bc](#), [98ab](#).

$\langle C \text{ KronSums Answer } 96a \rangle \equiv$

```
double *PQ_ans
◇
```

Fragment referenced in 77c, 83b, 87b, 95a, 97bc, 98ab, 100bcd, 101a, 103a, 104ab, 105ab.

$\langle \text{KronSums Integer } x \text{ } 96b \rangle \equiv$

```
if (SYMMETRIC) error("not implemented");
if (CENTER) error("not implemented");
if (TYPEOF(weights) == INTSXP) {
  if (TYPEOF(subset) == INTSXP) {
    C_XfactorKronSums_iweights_isubset(INTEGER(x), N, P, y, Q,
      INTEGER(weights), XLENGTH(weights) > 0, INTEGER(subset),
      offset, Nsubset, PQ_ans);
  } else {
    C_XfactorKronSums_iweights_dsubset(INTEGER(x), N, P, y, Q,
      INTEGER(weights), XLENGTH(weights) > 0, REAL(subset),
      offset, Nsubset, PQ_ans);
  }
} else {
  if (TYPEOF(subset) == INTSXP) {
    C_XfactorKronSums_dweights_isubset(INTEGER(x), N, P, y, Q,
      REAL(weights), XLENGTH(weights) > 0, INTEGER(subset),
      offset, Nsubset, PQ_ans);
  } else {
    C_XfactorKronSums_dweights_dsubset(INTEGER(x), N, P, y, Q,
      REAL(weights), XLENGTH(weights) > 0, REAL(subset),
      offset, Nsubset, PQ_ans);
  }
}
◇
```

Fragment referenced in 95b.

Uses: C\_XfactorKronSums\_dweights\_dsubset 100b, C\_XfactorKronSums\_dweights\_isubset 101a,  
C\_XfactorKronSums\_iweights\_dsubset 100c, C\_XfactorKronSums\_iweights\_isubset 100d, N 23bc, Nsubset 26b,  
offset 26c, P 24a, Q 24e, subset 26ade, weights 25b, x 23d, 24bc, y 24df, 25a.

$\langle \text{KronSums Double } x \text{ 97a} \rangle \equiv$

```

if (TYPEOF(weights) == INTSXP) {
  if (TYPEOF(subset) == INTSXP) {
    C_KronSums_iweights_isubset(REAL(x), N, P, y, Q, SYMMETRIC, centerx, centery, CENTER,
      INTEGER(weights), XLENGTH(weights) > 0, INTEGER(subset),
      offset, Nsubset, PQ_ans);
  } else {
    C_KronSums_iweights_dsubset(REAL(x), N, P, y, Q, SYMMETRIC, centerx, centery, CENTER,
      INTEGER(weights), XLENGTH(weights) > 0, REAL(subset),
      offset, Nsubset, PQ_ans);
  }
} else {
  if (TYPEOF(subset) == INTSXP) {
    C_KronSums_dweights_isubset(REAL(x), N, P, y, Q, SYMMETRIC, centerx, centery, CENTER,
      REAL(weights), XLENGTH(weights) > 0, INTEGER(subset),
      offset, Nsubset, PQ_ans);
  } else {
    C_KronSums_dweights_dsubset(REAL(x), N, P, y, Q, SYMMETRIC, centerx, centery, CENTER,
      REAL(weights), XLENGTH(weights) > 0, REAL(subset),
      offset, Nsubset, PQ_ans);
  }
}
}

```

Fragment referenced in 95b.

Uses: C\_KronSums\_dweights\_dsubset 97b, C\_KronSums\_dweights\_isubset 98b, C\_KronSums\_iweights\_dsubset 97c,  
 C\_KronSums\_iweights\_isubset 98a, N 23bc, Nsubset 26b, offset 26c, P 24a, Q 24e, subset 26ade, weights 25b,  
 x 23d, 24bc, y 24df, 25a.

$\langle \text{C\_KronSums\_dweights\_dsubset 97b} \rangle \equiv$

```

void C_KronSums_dweights_dsubset
(
   $\langle \text{C KronSums Input 95d} \rangle$ 
   $\langle \text{C real weights Input 25d} \rangle$ 
   $\langle \text{C real subset Input 26e} \rangle$ ,
   $\langle \text{C KronSums Answer 96a} \rangle$ 
) {
  double *s, *w;
   $\langle \text{KronSums Body 99} \rangle$ 
}

```

Fragment referenced in 93b.

Defines: C\_KronSums\_dweights\_dsubset 97a.

$\langle \text{C\_KronSums\_iweights\_dsubset 97c} \rangle \equiv$

```

void C_KronSums_iweights_dsubset
(
   $\langle \text{C KronSums Input 95d} \rangle$ 
   $\langle \text{C integer weights Input 25c} \rangle$ 
   $\langle \text{C real subset Input 26e} \rangle$ ,
   $\langle \text{C KronSums Answer 96a} \rangle$ 
) {
  double *s;
  int *w;
   $\langle \text{KronSums Body 99} \rangle$ 
}

```

Fragment referenced in 93b.

Defines: C\_KronSums\_iweights\_dsubset 97a.

$\langle C\_KronSums\_iweights\_isubset\ 98a \rangle \equiv$

```
void C_KronSums_iweights_isubset
(
   $\langle C\ KronSums\ Input\ 95d \rangle$ 
   $\langle C\ integer\ weights\ Input\ 25c \rangle$ 
   $\langle C\ integer\ subset\ Input\ 26d \rangle$ ,
   $\langle C\ KronSums\ Answer\ 96a \rangle$ 
) {
  int *s, *w;
   $\langle KronSums\ Body\ 99 \rangle$ 
}
◇
```

Fragment referenced in [93b](#).

Defines: `C_KronSums_iweights_isubset` [97a](#).

$\langle C\_KronSums\_dweights\_isubset\ 98b \rangle \equiv$

```
void C_KronSums_dweights_isubset
(
   $\langle C\ KronSums\ Input\ 95d \rangle$ 
   $\langle C\ real\ weights\ Input\ 25d \rangle$ 
   $\langle C\ integer\ subset\ Input\ 26d \rangle$ ,
   $\langle C\ KronSums\ Answer\ 96a \rangle$ 
) {
  int *s;
  double *w;
   $\langle KronSums\ Body\ 99 \rangle$ 
}
◇
```

Fragment referenced in [93b](#).

Defines: `C_KronSums_dweights_isubset` [97a](#).

⟨ *KronSums Body 99* ⟩ ≡

```
double *xx, *yy, cx = 0.0, cy = 0.0, *thisPQ_ans;
int idx;

for (int p = 0; p < P; p++) {
  for (int q = (SYMMETRIC ? p : 0); q < Q; q++) {
    /* SYMMETRIC is column-wise, default
       is row-wise (maybe need to change this) */
    if (SYMMETRIC) {
      idx = S(p, q, P);
    } else {
      idx = q * P + p;
    }
    PQ_ans[idx] = 0.0;
    thisPQ_ans = PQ_ans + idx;
    yy = y + N * q;
    xx = x + N * p;

    if (CENTER) {
      cx = centerx[p];
      cy = centery[q];
    }
    ⟨ init subset loop 88b ⟩
    ⟨ start subset loop 88c ⟩
    {
      xx = xx + diff;
      yy = yy + diff;
      if (HAS_WEIGHTS) {
        w = w + diff;
        if (CENTER) {
          thisPQ_ans[0] += (xx[0] - cx) * (yy[0] - cy) * w[0];
        } else {
          thisPQ_ans[0] += xx[0] * yy[0] * w[0];
        }
      } else {
        if (CENTER) {
          thisPQ_ans[0] += (xx[0] - cx) * (yy[0] - cy);
        } else {
          thisPQ_ans[0] += xx[0] * yy[0];
        }
      }
      ⟨ continue subset loop 89a ⟩
    }
    xx = xx + diff;
    yy = yy + diff;
    if (HAS_WEIGHTS) {
      w = w + diff;
      thisPQ_ans[0] += (xx[0] - cx) * (yy[0] - cy) * w[0];
    } else {
      thisPQ_ans[0] += (xx[0] - cx) * (yy[0] - cy);
    }
  }
}
}
◇
```

Fragment referenced in [97bc](#), [98ab](#).

Uses: HAS\_WEIGHTS [25cd](#), N [23bc](#), P [24a](#), Q [24e](#), S [21a](#), x [23d](#), [24bc](#), y [24df](#), [25a](#).

## Xfactor Kronecker Sums

$\langle C \text{ XfactorKronSums Input 100a} \rangle \equiv$

```
     $\langle C \text{ integer } x \text{ Input 24c} \rangle$   
     $\langle C \text{ real } y \text{ Input 24f} \rangle$   
     $\diamond$ 
```

Fragment referenced in [100bcd](#), [101a](#).

$\langle C\_XfactorKronSums\_dweights\_dsubset \text{ 100b} \rangle \equiv$

```
void C_XfactorKronSums_dweights_dsubset  
(  
     $\langle C \text{ XfactorKronSums Input 100a} \rangle$   
     $\langle C \text{ real weights Input 25d} \rangle$   
     $\langle C \text{ real subset Input 26e} \rangle$ ,  
     $\langle C \text{ KronSums Answer 96a} \rangle$   
) {  
    double *s, *w;  
     $\langle XfactorKronSums Body 101b \rangle$   
}  
 $\diamond$ 
```

Fragment referenced in [93b](#).

Defines: `C_XfactorKronSums_dweights_dsubset` [96b](#).

$\langle C\_XfactorKronSums\_iweights\_dsubset \text{ 100c} \rangle \equiv$

```
void C_XfactorKronSums_iweights_dsubset  
(  
     $\langle C \text{ XfactorKronSums Input 100a} \rangle$   
     $\langle C \text{ integer weights Input 25c} \rangle$   
     $\langle C \text{ real subset Input 26e} \rangle$ ,  
     $\langle C \text{ KronSums Answer 96a} \rangle$   
) {  
    double *s;  
    int *w;  
     $\langle XfactorKronSums Body 101b \rangle$   
}  
 $\diamond$ 
```

Fragment referenced in [93b](#).

Defines: `C_XfactorKronSums_iweights_dsubset` [96b](#).

$\langle C\_XfactorKronSums\_iweights\_isubset \text{ 100d} \rangle \equiv$

```
void C_XfactorKronSums_iweights_isubset  
(  
     $\langle C \text{ XfactorKronSums Input 100a} \rangle$   
     $\langle C \text{ integer weights Input 25c} \rangle$   
     $\langle C \text{ integer subset Input 26d} \rangle$ ,  
     $\langle C \text{ KronSums Answer 96a} \rangle$   
) {  
    int *s, *w;  
     $\langle XfactorKronSums Body 101b \rangle$   
}  
 $\diamond$ 
```

Fragment referenced in [93b](#).

Defines: `C_XfactorKronSums_iweights_isubset` [96b](#).

*< C\_XfactorKronSums\_dweights\_isubset 101a >* ≡

```
void C_XfactorKronSums_dweights_isubset
(
  < C XfactorKronSums Input 100a >
  < C real weights Input 25d >
  < C integer subset Input 26d >,
  < C KronSums Answer 96a >
) {
  int *s;
  double *w;
  < XfactorKronSums Body 101b >
}
◇
```

Fragment referenced in [93b](#).

Defines: `C_XfactorKronSums_dweights_isubset` [96b](#).

*< XfactorKronSums Body 101b >* ≡

```
int *xx, ixi;
double *yy;

for (int p = 0; p < mPQB(P, Q, 1); p++) PQ_ans[p] = 0.0;

for (int q = 0; q < Q; q++) {
  yy = y + N * q;
  xx = x;
  < init subset loop 88b >
  < start subset loop 88c >
  {
    xx = xx + diff;
    yy = yy + diff;
    ixi = xx[0] - 1;
    if (HAS_WEIGHTS) {
      w = w + diff;
      if (ixi >= 0)
        PQ_ans[ixi + q * P] += yy[0] * w[0];
    } else {
      if (ixi >= 0)
        PQ_ans[ixi + q * P] += yy[0];
    }
    < continue subset loop 89a >
  }
  xx = xx + diff;
  yy = yy + diff;
  ixi = xx[0] - 1;
  if (HAS_WEIGHTS) {
    w = w + diff;
    if (ixi >= 0)
      PQ_ans[ixi + q * P] += yy[0] * w[0];
  } else {
    if (ixi >= 0)
      PQ_ans[ixi + q * P] += yy[0];
  }
}
◇
```

Fragment referenced in [100bcd](#), [101a](#).

Uses: `HAS_WEIGHTS` [25cd](#), `mPQB` [132a](#), `N` [23bc](#), `P` [24a](#), `Q` [24e](#), `x` [23d](#), [24bc](#), `y` [24df](#), [25a](#).

## Permuted Kronecker Sums

```
> a0 <- colSums(x[subset, r1] * y[subsety, r2])
> a1 <- .Call(libcoin:::R_KronSums_Permutation, x, P, y, subset, subsety)
> a2 <- .Call(libcoin:::R_KronSums_Permutation, x, P, y, as.double(subset), as.double(subsety))
> stopifnot(isequal(a0, a1) && isequal(a0, a2))
> a0 <- as.vector(colSums(Xfactor[subset, r1Xfactor] * y[subsety, r2Xfactor]))
> a1 <- .Call(libcoin:::R_KronSums_Permutation, ix, Lx, y, subset, subsety)
> a2 <- .Call(libcoin:::R_KronSums_Permutation, ix, Lx, y, as.double(subset), as.double(subsety))
> stopifnot(isequal(a0, a1) && isequal(a0, a2))
```

*< R\_KronSums\_Permutation Prototype 102a >* ≡

```
SEXP R_KronSums_Permutation
(
  < R x Input 23d >
  SEXP P,
  < R y Input 24d >
  < R subset Input 26a >,
  SEXP subsety
)
◇
```

Fragment referenced in [22b](#), [102b](#).

Uses: P [24a](#), R\_KronSums\_Permutation [102b](#).

*< R\_KronSums\_Permutation 102b >* ≡

```
< R_KronSums_Permutation Prototype 102a >
{
  SEXP ans;
  < C integer Q Input 24e >;
  < C integer N Input 23c >;
  < C integer Nsubset Input 26b >;

  Q = NCOL(y);
  N = XLENGTH(y) / Q;
  Nsubset = XLENGTH(subset);

  PROTECT(ans = allocVector(REALSXP, INTEGER(P)[0] * Q));
  RC_KronSums_Permutation(x, N, INTEGER(P)[0], REAL(y), Q, subset, Offset0, Nsubset,
                        subsety, REAL(ans));
  UNPROTECT(1);
  return(ans);
}
◇
```

Fragment referenced in [93b](#).

Defines: R\_KronSums\_Permutation [102a](#), [152c](#), [153](#).

Uses: N [23bc](#), NCOL [130c](#), Nsubset [26b](#), Offset0 [21b](#), P [24a](#), Q [24e](#), RC\_KronSums\_Permutation [103b](#), subset [26ade](#), x [23d](#), [24bc](#), y [24df](#), [25a](#).

$\langle RC\_KronSums\_Permutation\ Prototype\ 103a \rangle \equiv$

```
void RC_KronSums_Permutation
(
   $\langle R\ x\ Input\ 23d \rangle$ 
   $\langle C\ integer\ N\ Input\ 23c \rangle$ ,
   $\langle C\ integer\ P\ Input\ 24a \rangle$ ,
   $\langle C\ real\ y\ Input\ 24f \rangle$ 
   $\langle R\ subset\ Input\ 26a \rangle$ ,
   $\langle C\ subset\ range\ Input\ 26c \rangle$ ,
  SEXP subsety,
   $\langle C\ KronSums\ Answer\ 96a \rangle$ 
)
```

Fragment referenced in [103b](#).

Uses: [RC\\_KronSums\\_Permutation 103b](#).

$\langle RC\_KronSums\_Permutation\ 103b \rangle \equiv$

```
 $\langle RC\_KronSums\_Permutation\ Prototype\ 103a \rangle$ 
{
  if (typeof(x) == INTSXP) {
    if (typeof(subset) == INTSXP) {
      C_XfactorKronSums_Permutation_isubset(INTEGER(x), N, P, y, Q,
                                           INTEGER(subset), offset, Nsubset,
                                           INTEGER(subsety), PQ_ans);
    } else {
      C_XfactorKronSums_Permutation_dsubset(INTEGER(x), N, P, y, Q,
                                           REAL(subset), offset, Nsubset,
                                           REAL(subsety), PQ_ans);
    }
  } else {
    if (typeof(subset) == INTSXP) {
      C_KronSums_Permutation_isubset(REAL(x), N, P, y, Q,
                                     INTEGER(subset), offset, Nsubset,
                                     INTEGER(subsety), PQ_ans);
    } else {
      C_KronSums_Permutation_dsubset(REAL(x), N, P, y, Q,
                                     REAL(subset), offset, Nsubset,
                                     REAL(subsety), PQ_ans);
    }
  }
}
```

Fragment referenced in [93b](#).

Defines: [RC\\_KronSums\\_Permutation 37](#), [102b](#), [103a](#).

Uses: [C\\_KronSums\\_Permutation\\_dsubset 104a](#), [C\\_KronSums\\_Permutation\\_isubset 104b](#),  
[C\\_XfactorKronSums\\_Permutation\\_dsubset 105a](#), [C\\_XfactorKronSums\\_Permutation\\_isubset 105b](#), [N 23bc](#),  
[Nsubset 26b](#), [offset 26c](#), [P 24a](#), [Q 24e](#), [subset 26ade](#), [x 23d](#), [24bc](#), [y 24df](#), [25a](#).

$\langle C\_KronSums\_Permutation\_dsubset\ 104a \rangle \equiv$

```
void C_KronSums_Permutation_dsubset
(
     $\langle C\ real\ x\ Input\ 24b \rangle$ 
     $\langle C\ real\ y\ Input\ 24f \rangle$ 
     $\langle C\ real\ subset\ Input\ 26e \rangle$ ,
    double *subsety,
     $\langle C\ KronSums\ Answer\ 96a \rangle$ 
) {
     $\langle KronSums\ Permutation\ Body\ 104c \rangle$ 
}
◇
```

Fragment referenced in [93b](#).

Defines: `C_KronSums_Permutation_dsubset` [103b](#).

$\langle C\_KronSums\_Permutation\_isubset\ 104b \rangle \equiv$

```
void C_KronSums_Permutation_isubset
(
     $\langle C\ real\ x\ Input\ 24b \rangle$ 
     $\langle C\ real\ y\ Input\ 24f \rangle$ 
     $\langle C\ integer\ subset\ Input\ 26d \rangle$ ,
    int *subsety,
     $\langle C\ KronSums\ Answer\ 96a \rangle$ 
) {
     $\langle KronSums\ Permutation\ Body\ 104c \rangle$ 
}
◇
```

Fragment referenced in [93b](#).

Defines: `C_KronSums_Permutation_isubset` [103b](#).

Because `subset` might not be ordered (in the presence of blocks) we have to go through all elements explicitly here.

$\langle KronSums\ Permutation\ Body\ 104c \rangle \equiv$

```
R_xlen_t qP, qN, pN, qPp;

for (int q = 0; q < Q; q++) {
    qN = q * N;
    qP = q * P;
    for (int p = 0; p < P; p++) {
        qPp = qP + p;
        PQ_ans[qPp] = 0.0;
        pN = p * N;
        for (R_xlen_t i = offset; i < Nsubset; i++)
            PQ_ans[qPp] += y[qN + (R_xlen_t) subsety[i] - 1] *
                x[pN + (R_xlen_t) subset[i] - 1];
    }
}
◇
```

Fragment referenced in [104ab](#).

Uses: `N` [23bc](#), `Nsubset` [26b](#), `offset` [26c](#), `P` [24a](#), `Q` [24e](#), `subset` [26ade](#), `x` [23d](#), [24bc](#), `y` [24df](#), [25a](#).

## Xfactor Permuted Kronecker Sums

$\langle C\_XfactorKronSums\_Permutation\_dsubset\ 105a \rangle \equiv$

```
void C_XfactorKronSums_Permutation_dsubset
(
     $\langle C\ integer\ x\ Input\ 24c \rangle$ 
     $\langle C\ real\ y\ Input\ 24f \rangle$ 
     $\langle C\ real\ subset\ Input\ 26e \rangle$ ,
    double *subsety,
     $\langle C\ KronSums\ Answer\ 96a \rangle$ 
) {
     $\langle XfactorKronSums\ Permutation\ Body\ 105c \rangle$ 
}
◇
```

Fragment referenced in [93b](#).

Defines: `C_XfactorKronSums_Permutation_dsubset` [103b](#).

$\langle C\_XfactorKronSums\_Permutation\_isubset\ 105b \rangle \equiv$

```
void C_XfactorKronSums_Permutation_isubset
(
     $\langle C\ integer\ x\ Input\ 24c \rangle$ 
     $\langle C\ real\ y\ Input\ 24f \rangle$ 
     $\langle C\ integer\ subset\ Input\ 26d \rangle$ ,
    int *subsety,
     $\langle C\ KronSums\ Answer\ 96a \rangle$ 
) {
     $\langle XfactorKronSums\ Permutation\ Body\ 105c \rangle$ 
}
◇
```

Fragment referenced in [93b](#).

Defines: `C_XfactorKronSums_Permutation_isubset` [103b](#).

$\langle XfactorKronSums\ Permutation\ Body\ 105c \rangle \equiv$

```
R_xlen_t qP, qN;

for (int p = 0; p < mPQB(P, Q, 1); p++) PQ_ans[p] = 0.0;

for (int q = 0; q < Q; q++) {
    qP = q * P;
    qN = q * N;
    for (R_xlen_t i = offset; i < Nsubset; i++)
        PQ_ans[x[(R_xlen_t) subset[i] - 1] - 1 + qP] += y[qN + (R_xlen_t) subsety[i] - 1];
}
◇
```

Fragment referenced in [105ab](#).

Uses: `mPQB` [132a](#), `N` [23bc](#), `Nsubset` [26b](#), `offset` [26c](#), `P` [24a](#), `Q` [24e](#), `subset` [26ade](#), `x` [23d](#), `24bc`, `y` [24df](#), [25a](#).

### 3.9.3 Column Sums

*< colSums 106a >* ≡

```
< C_colSums_dweights_dsubset 108b >
< C_colSums_iveights_dsubset 108c >
< C_colSums_iveights_isset 108d >
< C_colSums_dweights_isset 109a >
< RC_colSums 107b >
< R_colSums 106c >
◇
```

Fragment referenced in [23a](#).

```
> a0 <- colSums(x[subset, ] * weights[subset])
> a1 <- .Call(libcoin:::R_colSums, x, weights, subset)
> a2 <- .Call(libcoin:::R_colSums, x, as.double(weights), as.double(subset))
> a3 <- .Call(libcoin:::R_colSums, x, weights, as.double(subset))
> a4 <- .Call(libcoin:::R_colSums, x, as.double(weights), subset)
> stopifnot(isequal(a0, a1) && isequal(a0, a2) &&
+           isequal(a0, a3) && isequal(a0, a4))
```

*< R\_colSums Prototype 106b >* ≡

```
SEXP R_colSums
(
  < R x Input 23d >
  < R weights Input 25b >,
  < R subset Input 26a >
)
◇
```

Fragment referenced in [22b](#), [106c](#).

Uses: [R\\_colSums 106c](#).

*< R\_colSums 106c >* ≡

```
< R_colSums Prototype 106b >
{
  SEXP ans;
  int P;
  < C integer N Input 23c >;
  < C integer Nsubset Input 26b >;
  double center;

  P = NCOL(x);
  N = XLENGTH(x) / P;
  Nsubset = XLENGTH(subset);

  PROTECT(ans = allocVector(REALSXP, P));
  RC_colSums(REAL(x), N, P, Power1, &center, !DoCenter, weights, subset, Offset0,
             Nsubset, REAL(ans));
  UNPROTECT(1);
  return(ans);
}
◇
```

Fragment referenced in [106a](#).

Defines: [R\\_colSums 106b](#), [152c](#), [153](#).

Uses: [DoCenter 21b](#), [N 23bc](#), [NCOL 130c](#), [Nsubset 26b](#), [Offset0 21b](#), [P 24a](#), [Power1 21b](#), [RC\\_colSums 107b](#), [subset 26ade](#), [weights 25b](#), [weights, 25cd](#), [x 23d](#), [24bc](#).

*< RC\_colSums Prototype 107a >* ≡

```
void RC_colSums
(
    < C colSums Input 107c >
    < R weights Input 25b >,
    < R subset Input 26a >,
    < C subset range Input 26c >,
    < C colSums Answer 108a >
)
◇
```

Fragment referenced in [107b](#).

Uses: [RC\\_colSums 107b](#).

*< RC\_colSums 107b >* ≡

```
< RC_colSums Prototype 107a >
{
    if (TYPEOF(weights) == INTSXP) {
        if (TYPEOF(subset) == INTSXP) {
            C_colSums_iweights_isubset(x, N, P, power, centerx, CENTER,
                                       INTEGER(weights), XLENGTH(weights) > 0, INTEGER(subset),
                                       offset, Nsubset, P_ans);
        } else {
            C_colSums_iweights_dsubset(x, N, P, power, centerx, CENTER,
                                       INTEGER(weights), XLENGTH(weights) > 0, REAL(subset),
                                       offset, Nsubset, P_ans);
        }
    } else {
        if (TYPEOF(subset) == INTSXP) {
            C_colSums_dweights_isubset(x, N, P, power, centerx, CENTER,
                                       REAL(weights), XLENGTH(weights) > 0, INTEGER(subset),
                                       offset, Nsubset, P_ans);
        } else {
            C_colSums_dweights_dsubset(x, N, P, power, centerx, CENTER,
                                       REAL(weights), XLENGTH(weights) > 0, REAL(subset),
                                       offset, Nsubset, P_ans);
        }
    }
}
◇
```

Fragment referenced in [106a](#).

Defines: [RC\\_colSums 81c, 84a, 85b, 88a, 106c, 107a](#).

Uses: [C\\_colSums\\_dweights\\_dsubset 108b](#), [C\\_colSums\\_dweights\\_isubset 109a](#), [C\\_colSums\\_iweights\\_dsubset 108c](#),  
[C\\_colSums\\_iweights\\_isubset 108d](#), [N 23bc](#), [Nsubset 26b](#), [offset 26c](#), [P 24a](#), [subset 26ade](#), [weights 25b](#), [x 23d](#),  
[24bc](#).

*< C colSums Input 107c >* ≡

```
< C real x Input 24b >
const int power,
double *centerx,
const int CENTER,
◇
```

Fragment referenced in [107a](#), [108bcd](#), [109a](#).

*< C colSums Answer 108a >* ≡

```
double *P_ans
◇
```

Fragment referenced in [81b](#), [107a](#), [108bcd](#), [109a](#).

*< C\_colSums\_dweights\_dsubset 108b >* ≡

```
void C_colSums_dweights_dsubset
(
  < C colSums Input 107c >
  < C real weights Input 25d >
  < C real subset Input 26e >,
  < C colSums Answer 108a >
) {
  double *s, *w;
  < colSums Body 109b >
}
◇
```

Fragment referenced in [106a](#).

Defines: `C_colSums_dweights_dsubset` [107b](#).

*< C\_colSums\_ieweights\_dsubset 108c >* ≡

```
void C_colSums_ieweights_dsubset
(
  < C colSums Input 107c >
  < C integer weights Input 25c >
  < C real subset Input 26e >,
  < C colSums Answer 108a >
) {
  double *s;
  int *w;
  < colSums Body 109b >
}
◇
```

Fragment referenced in [106a](#).

Defines: `C_colSums_ieweights_dsubset` [107b](#).

*< C\_colSums\_ieweights\_isubset 108d >* ≡

```
void C_colSums_ieweights_isubset
(
  < C colSums Input 107c >
  < C integer weights Input 25c >
  < C integer subset Input 26d >,
  < C colSums Answer 108a >
) {
  int *s, *w;
  < colSums Body 109b >
}
◇
```

Fragment referenced in [106a](#).

Defines: `C_colSums_ieweights_isubset` [107b](#).

*< C\_colSums\_dweights\_isubset 109a >* ≡

```
void C_colSums_dweights_isubset
(
  < C_colSums Input 107c >
  < C_real_weights Input 25d >
  < C_integer_subset Input 26d >,
  < C_colSums Answer 108a >
) {
  int *s;
  double *w;
  < colSums Body 109b >
}
◇
```

Fragment referenced in 106a.

Defines: C\_colSums\_dweights\_isubset 107b.

*< colSums Body 109b >* ≡

```
double *xx, cx = 0.0;

for (int p = 0; p < P; p++) {
  P_ans[0] = 0.0;
  xx = x + N * p;
  if (CENTER) {
    cx = centerx[p];
  }
  < init_subset loop 88b >
  < start_subset loop 88c >
  {
    xx = xx + diff;
    if (HAS_WEIGHTS) {
      w = w + diff;
      P_ans[0] += pow(xx[0] - cx, power) * w[0];
    } else {
      P_ans[0] += pow(xx[0] - cx, power);
    }
    < continue_subset loop 89a >
  }
  xx = xx + diff;
  if (HAS_WEIGHTS) {
    w = w + diff;
    P_ans[0] += pow(xx[0] - cx, power) * w[0];
  } else {
    P_ans[0] += pow(xx[0] - cx, power);
  }
  P_ans++;
}
◇
```

Fragment referenced in 108bcd, 109a.

Uses: HAS\_WEIGHTS 25cd, N 23bc, P 24a, x 23d, 24bc.

### 3.9.4 Tables

#### OneTable Sums

⟨ Tables 110a ⟩ ≡

```
⟨ C_OneTableSums_dweights_dsubset 112d ⟩
⟨ C_OneTableSums_iveights_dsubset 113a ⟩
⟨ C_OneTableSums_iveights_isubset 113b ⟩
⟨ C_OneTableSums_dweights_isubset 113c ⟩
⟨ RC_OneTableSums 112a ⟩
⟨ R_OneTableSums 111a ⟩
⟨ C_TwoTableSums_dweights_dsubset 117a ⟩
⟨ C_TwoTableSums_iveights_dsubset 117b ⟩
⟨ C_TwoTableSums_iveights_isubset 117c ⟩
⟨ C_TwoTableSums_dweights_isubset 118a ⟩
⟨ RC_TwoTableSums 116a ⟩
⟨ R_TwoTableSums 115a ⟩
⟨ C_ThreeTableSums_dweights_dsubset 121b ⟩
⟨ C_ThreeTableSums_iveights_dsubset 121c ⟩
⟨ C_ThreeTableSums_iveights_isubset 121d ⟩
⟨ C_ThreeTableSums_dweights_isubset 122a ⟩
⟨ RC_ThreeTableSums 120b ⟩
⟨ R_ThreeTableSums 119b ⟩
```

◇

Fragment referenced in [23a](#).

```
> a0 <- as.vector(xtabs(weights ~ ixf, subset = subset))
> a1 <- ctabs(ix, weights = weights, subset = subset)[-1]
> a2 <- ctabs(ix, weights = as.double(weights), subset = as.double(subset))[-1]
> a3 <- ctabs(ix, weights = weights, subset = as.double(subset))[-1]
> a4 <- ctabs(ix, weights = as.double(weights), subset = subset)[-1]
> stopifnot(isequal(a0, a1) && isequal(a0, a2) &&
+           isequal(a0, a3) && isequal(a0, a4))
```

⟨ R\_OneTableSums Prototype 110b ⟩ ≡

```
SEXP R_OneTableSums
(
  ⟨ R x Input 23d ⟩
  ⟨ R weights Input 25b ⟩,
  ⟨ R subset Input 26a ⟩
)
```

◇

Fragment referenced in [22b](#), [111a](#).

Uses: [R\\_OneTableSums 111a](#).

$\langle R\_OneTableSums\ 111a \rangle \equiv$

```
 $\langle R\_OneTableSums\ Prototype\ 110b \rangle$ 
{
    SEXP ans;
     $\langle C\ integer\ N\ Input\ 23c \rangle$ ;
     $\langle C\ integer\ Nsubset\ Input\ 26b \rangle$ ;
    int P;

    N = XLENGTH(x);
    Nsubset = XLENGTH(subset);
    P = NLEVELS(x) + 1;

    PROTECT(ans = allocVector(REALSXP, P));
    RC_OneTableSums(INTEGER(x), N, P, weights, subset,
                    Offset0, Nsubset, REAL(ans));
    UNPROTECT(1);
    return(ans);
}
◇
```

Fragment referenced in 110a.

Defines: R\_OneTableSums 15b, 110b, 123c, 152c, 153.

Uses: N 23bc, NLEVELS 131a, Nsubset 26b, Offset0 21b, P 24a, RC\_OneTableSums 112a, subset 26ade, weights 25b, weights, 25cd, x 23d, 24bc.

$\langle RC\_OneTableSums\ Prototype\ 111b \rangle \equiv$

```
void RC_OneTableSums
(
     $\langle C\ OneTableSums\ Input\ 112b \rangle$ 
     $\langle R\ weights\ Input\ 25b \rangle$ ,
     $\langle R\ subset\ Input\ 26a \rangle$ ,
     $\langle C\ subset\ range\ Input\ 26c \rangle$ ,
     $\langle C\ OneTableSums\ Answer\ 112c \rangle$ 
)
◇
```

Fragment referenced in 112a.

Uses: RC\_OneTableSums 112a.

$\langle RC\_OneTableSums\ 112a \rangle \equiv$

```
 $\langle RC\_OneTableSums\ Prototype\ 111b \rangle$ 
{
  if (TYPEOF(weights) == INTSXP) {
    if (TYPEOF(subset) == INTSXP) {
      C_OneTableSums_iweights_isubset(x, N, P,
                                     INTEGER(weights), XLENGTH(weights) > 0, INTEGER(subset),
                                     offset, Nsubset, P_ans);
    } else {
      C_OneTableSums_iweights_dsubset(x, N, P,
                                     INTEGER(weights), XLENGTH(weights) > 0, REAL(subset),
                                     offset, Nsubset, P_ans);
    }
  } else {
    if (TYPEOF(subset) == INTSXP) {
      C_OneTableSums_dweights_isubset(x, N, P,
                                     REAL(weights), XLENGTH(weights) > 0, INTEGER(subset),
                                     offset, Nsubset, P_ans);
    } else {
      C_OneTableSums_dweights_dsubset(x, N, P,
                                     REAL(weights), XLENGTH(weights) > 0, REAL(subset),
                                     offset, Nsubset, P_ans);
    }
  }
}
◇
```

Fragment referenced in [110a](#).

Defines: [RC\\_OneTableSums 34a](#), [37](#), [85b](#), [111ab](#).

Uses: [C\\_OneTableSums\\_dweights\\_dsubset 112d](#), [C\\_OneTableSums\\_dweights\\_isubset 113c](#),  
[C\\_OneTableSums\\_iweights\\_dsubset 113a](#), [C\\_OneTableSums\\_iweights\\_isubset 113b](#), [N 23bc](#), [Nsubset 26b](#),  
[offset 26c](#), [P 24a](#), [subset 26ade](#), [weights 25b](#), [x 23d](#), [24bc](#).

$\langle C\ OneTableSums\ Input\ 112b \rangle \equiv$

```
 $\langle C\ integer\ x\ Input\ 24c \rangle$ 
◇
```

Fragment referenced in [111b](#), [112d](#), [113abc](#).

$\langle C\ OneTableSums\ Answer\ 112c \rangle \equiv$

```
double *P_ans
◇
```

Fragment referenced in [85a](#), [111b](#), [112d](#), [113abc](#).

$\langle C\_OneTableSums\_dweights\_dsubset\ 112d \rangle \equiv$

```
void C_OneTableSums_dweights_dsubset
(
   $\langle C\_OneTableSums\_Input\ 112b \rangle$ 
   $\langle C\_real\ weights\ Input\ 25d \rangle$ 
   $\langle C\_real\ subset\ Input\ 26e \rangle$ ,
   $\langle C\_OneTableSums\_Answer\ 112c \rangle$ 
) {
  double *s, *w;
   $\langle OneTableSums\ Body\ 114a \rangle$ 
}
◇
```

Fragment referenced in 110a.

Defines: C\_OneTableSums\_dweights\_dsubset 112a.

$\langle C\_OneTableSums\_iweights\_dsubset\ 113a \rangle \equiv$

```
void C_OneTableSums_iweights_dsubset
(
   $\langle C\_OneTableSums\_Input\ 112b \rangle$ 
   $\langle C\_integer\ weights\ Input\ 25c \rangle$ 
   $\langle C\_real\ subset\ Input\ 26e \rangle$ ,
   $\langle C\_OneTableSums\_Answer\ 112c \rangle$ 
) {
  double *s;
  int *w;
   $\langle OneTableSums\ Body\ 114a \rangle$ 
}
◇
```

Fragment referenced in 110a.

Defines: C\_OneTableSums\_iweights\_dsubset 112a.

$\langle C\_OneTableSums\_iweights\_isubset\ 113b \rangle \equiv$

```
void C_OneTableSums_iweights_isubset
(
   $\langle C\_OneTableSums\_Input\ 112b \rangle$ 
   $\langle C\_integer\ weights\ Input\ 25c \rangle$ 
   $\langle C\_integer\ subset\ Input\ 26d \rangle$ ,
   $\langle C\_OneTableSums\_Answer\ 112c \rangle$ 
) {
  int *s, *w;
   $\langle OneTableSums\ Body\ 114a \rangle$ 
}
◇
```

Fragment referenced in 110a.

Defines: C\_OneTableSums\_iweights\_isubset 112a.

*< C\_OneTableSums\_dweights\_isubset 113c >* ≡

```
void C_OneTableSums_dweights_isubset
(
  < C_OneTableSums Input 112b >
  < C_real_weights Input 25d >
  < C_integer_subset Input 26d >,
  < C_OneTableSums Answer 112c >
) {
  int *s;
  double *w;
  < OneTableSums Body 114a >
}
◇
```

Fragment referenced in 110a.

Defines: C\_OneTableSums\_dweights\_isubset 112a.

*< OneTableSums Body 114a >* ≡

```
int *xx;

for (int p = 0; p < P; p++) P_ans[p] = 0.0;

xx = x;
< init_subset_loop 88b >
< start_subset_loop 88c >
{
  xx = xx + diff;
  if (HAS_WEIGHTS) {
    w = w + diff;
    P_ans[xx[0]] += (double) w[0];
  } else {
    P_ans[xx[0]]++;
  }
  < continue_subset_loop 89a >
}
xx = xx + diff;
if (HAS_WEIGHTS) {
  w = w + diff;
  P_ans[xx[0]] += w[0];
} else {
  P_ans[xx[0]]++;
}
◇
```

Fragment referenced in 112d, 113abc.

Uses: HAS\_WEIGHTS 25cd, P 24a, x 23d, 24bc.

## TwoTable Sums

```
> a0 <- xtabs(weights ~ ixf + iyf, subset = subset)
> class(a0) <- "matrix"
> dimnames(a0) <- NULL
> attributes(a0)$call <- NULL
> a1 <- ctabs(ix, iy, weights = weights, subset = subset)[-1, -1]
> a2 <- ctabs(ix, iy, weights = as.double(weights),
+           subset = as.double(subset))[-1, -1]
> a3 <- ctabs(ix, iy, weights = weights, subset = as.double(subset))[-1, -1]
> a4 <- ctabs(ix, iy, weights = as.double(weights), subset = subset)[-1, -1]
> stopifnot(isequal(a0, a1) && isequal(a0, a2) &&
+           isequal(a0, a3) && isequal(a0, a4))
```

$\langle R\_TwoTableSums Prototype 114b \rangle \equiv$

```
SEXP R_TwoTableSums
(
   $\langle R x Input 23d \rangle$ 
   $\langle R y Input 24d \rangle$ 
   $\langle R weights Input 25b \rangle$ ,
   $\langle R subset Input 26a \rangle$ 
)
◇
```

Fragment referenced in [22b](#), [115a](#).  
Uses: [R\\_TwoTableSums 115a](#).

$\langle R\_TwoTableSums 115a \rangle \equiv$

```
 $\langle R\_TwoTableSums Prototype 114b \rangle$ 
{
  SEXP ans, dim;
   $\langle C integer N Input 23c \rangle$ ;
   $\langle C integer Nsubset Input 26b \rangle$ ;
  int P, Q;

  N = XLENGTH(x);
  Nsubset = XLENGTH(subset);
  P = NLEVELS(x) + 1;
  Q = NLEVELS(y) + 1;

  PROTECT(ans = allocVector(REALSXP, mPQB(P, Q, 1)));
  PROTECT(dim = allocVector(INTSXP, 2));
  INTEGER(dim)[0] = P;
  INTEGER(dim)[1] = Q;
  dimgets(ans, dim);
  RC_TwoTableSums(INTEGER(x), N, P, INTEGER(y), Q,
                  weights, subset, Offset0, Nsubset, REAL(ans));
  UNPROTECT(2);
  return(ans);
}
◇
```

Fragment referenced in [110a](#).  
Defines: [R\\_TwoTableSums 15b](#), [114b](#), [152c](#), [153](#).  
Uses: [mPQB 132a](#), [N 23bc](#), [NLEVELS 131a](#), [Nsubset 26b](#), [Offset0 21b](#), [P 24a](#), [Q 24c](#), [RC\\_TwoTableSums 116a](#), [subset 26ade](#),  
[weights 25b](#), [weights, 25cd](#), [x 23d](#), [24bc](#), [y 24df](#), [25a](#).

$\langle RC\_TwoTableSums Prototype 115b \rangle \equiv$

```
void RC_TwoTableSums
(
   $\langle C TwoTableSums Input 116b \rangle$ 
   $\langle R weights Input 25b \rangle$ ,
   $\langle R subset Input 26a \rangle$ ,
   $\langle C subset range Input 26c \rangle$ ,
   $\langle C TwoTableSums Answer 116c \rangle$ 
)
◇
```

Fragment referenced in [116a](#).  
Uses: [RC\\_TwoTableSums 116a](#).

*< RC\_TwoTableSums 116a >* ≡

*< RC\_TwoTableSums Prototype 115b >*

```
{
  if (typeof(weights) == INTSXP) {
    if (typeof(subset) == INTSXP) {
      C_TwoTableSums_iweights_isubset(x, N, P, y, Q,
                                     INTEGER(weights), XLENGTH(weights) > 0, INTEGER(subset),
                                     offset, Nsubset, PQ_ans);
    } else {
      C_TwoTableSums_iweights_dsubset(x, N, P, y, Q,
                                     INTEGER(weights), XLENGTH(weights) > 0, REAL(subset),
                                     offset, Nsubset, PQ_ans);
    }
  } else {
    if (typeof(subset) == INTSXP) {
      C_TwoTableSums_dweights_isubset(x, N, P, y, Q,
                                     REAL(weights), XLENGTH(weights) > 0, INTEGER(subset),
                                     offset, Nsubset, PQ_ans);
    } else {
      C_TwoTableSums_dweights_dsubset(x, N, P, y, Q,
                                     REAL(weights), XLENGTH(weights) > 0, REAL(subset),
                                     offset, Nsubset, PQ_ans);
    }
  }
}
```

Fragment referenced in [110a](#).

Defines: [RC\\_TwoTableSums 41a](#), [115ab](#).

Uses: [C\\_TwoTableSums\\_dweights\\_dsubset 117a](#), [C\\_TwoTableSums\\_dweights\\_isubset 118a](#),  
[C\\_TwoTableSums\\_iweights\\_dsubset 117b](#), [C\\_TwoTableSums\\_iweights\\_isubset 117c](#), [N 23bc](#), [Nsubset 26b](#),  
[offset 26c](#), [P 24a](#), [Q 24e](#), [subset 26ade](#), [weights 25b](#), [x 23d](#), [24bc](#), [y 24df](#), [25a](#).

*< C\_TwoTableSums Input 116b >* ≡

*< C integer x Input 24c >*

*< C integer y Input 25a >*

◇

Fragment referenced in [115b](#), [117abc](#), [118a](#).

*< C\_TwoTableSums Answer 116c >* ≡

double \*PQ\_ans

◇

Fragment referenced in [115b](#), [117abc](#), [118a](#).

$\langle C\_TwoTableSums\_dweights\_dsubset\ 117a \rangle \equiv$

```
void C_TwoTableSums_dweights_dsubset
(
   $\langle C\ TwoTableSums\ Input\ 116b \rangle$ 
   $\langle C\ real\ weights\ Input\ 25d \rangle$ 
   $\langle C\ real\ subset\ Input\ 26e \rangle$ ,
   $\langle C\ TwoTableSums\ Answer\ 116c \rangle$ 
) {
  double *s, *w;
   $\langle TwoTableSums\ Body\ 118b \rangle$ 
}
◇
```

Fragment referenced in 110a.

Defines: C\_TwoTableSums\_dweights\_dsubset 116a.

$\langle C\_TwoTableSums\_iweights\_dsubset\ 117b \rangle \equiv$

```
void C_TwoTableSums_iweights_dsubset
(
   $\langle C\ TwoTableSums\ Input\ 116b \rangle$ 
   $\langle C\ integer\ weights\ Input\ 25c \rangle$ 
   $\langle C\ real\ subset\ Input\ 26e \rangle$ ,
   $\langle C\ TwoTableSums\ Answer\ 116c \rangle$ 
) {
  double *s;
  int *w;
   $\langle TwoTableSums\ Body\ 118b \rangle$ 
}
◇
```

Fragment referenced in 110a.

Defines: C\_TwoTableSums\_iweights\_dsubset 116a.

$\langle C\_TwoTableSums\_iweights\_isubset\ 117c \rangle \equiv$

```
void C_TwoTableSums_iweights_isubset
(
   $\langle C\ TwoTableSums\ Input\ 116b \rangle$ 
   $\langle C\ integer\ weights\ Input\ 25c \rangle$ 
   $\langle C\ integer\ subset\ Input\ 26d \rangle$ ,
   $\langle C\ TwoTableSums\ Answer\ 116c \rangle$ 
) {
  int *s, *w;
   $\langle TwoTableSums\ Body\ 118b \rangle$ 
}
◇
```

Fragment referenced in 110a.

Defines: C\_TwoTableSums\_iweights\_isubset 116a.

$\langle C\_TwoTableSums\_dweights\_isubset\ 118a \rangle \equiv$

```
void C_TwoTableSums_dweights_isubset
(
   $\langle C\ TwoTableSums\ Input\ 116b \rangle$ 
   $\langle C\ real\ weights\ Input\ 25d \rangle$ 
   $\langle C\ integer\ subset\ Input\ 26d \rangle$ ,
   $\langle C\ TwoTableSums\ Answer\ 116c \rangle$ 
) {
  int *s;
  double *w;
   $\langle TwoTableSums\ Body\ 118b \rangle$ 
}
◇
```

Fragment referenced in 110a.

Defines: C\_TwoTableSums\_dweights\_isubset 116a.

$\langle TwoTableSums\ Body\ 118b \rangle \equiv$

```
int *xx, *yy;

for (int p = 0; p < Q * P; p++) PQ_ans[p] = 0.0;

yy = y;
xx = x;
 $\langle init\ subset\ loop\ 88b \rangle$ 
 $\langle start\ subset\ loop\ 88c \rangle$ 
{
  xx = xx + diff;
  yy = yy + diff;
  if (HAS_WEIGHTS) {
    w = w + diff;
    PQ_ans[yy[0] * P + xx[0]] += (double) w[0];
  } else {
    PQ_ans[yy[0] * P + xx[0]]++;
  }
   $\langle continue\ subset\ loop\ 89a \rangle$ 
}
xx = xx + diff;
yy = yy + diff;
if (HAS_WEIGHTS) {
  w = w + diff;
  PQ_ans[yy[0] * P + xx[0]] += w[0];
} else {
  PQ_ans[yy[0] * P + xx[0]]++;
}
}
◇
```

Fragment referenced in 117abc, 118a.

Uses: HAS\_WEIGHTS 25cd, P 24a, Q 24e, x 23d, 24bc, y 24df, 25a.

### ThreeTable Sums

```
> a0 <- xtabs(weights ~ ixf + iyf + block, subset = subset)
> class(a0) <- "array"
> dimnames(a0) <- NULL
> attributes(a0)$call <- NULL
> a1 <- ctabs(ix, iy, block, weights, subset)[-1, -1,]
> a2 <- ctabs(ix, iy, block, as.double(weights), as.double(subset))[-1, -1,]
> a3 <- ctabs(ix, iy, block, weights, as.double(subset))[-1, -1,]
> a4 <- ctabs(ix, iy, block, as.double(weights), subset)[-1, -1,]
```

```
> stopifnot(isequal(a0, a1) && isequal(a0, a2) &&
+           isequal(a0, a3) && isequal(a0, a4))
```

*< R\_ThreeTableSums Prototype 119a >* ≡

```
SEXP R_ThreeTableSums
(
  < R x Input 23d >
  < R y Input 24d >
  < R block Input 26f >,
  < R weights Input 25b >,
  < R subset Input 26a >
)
◇
```

Fragment referenced in [22b](#), [119b](#).

Uses: [R\\_ThreeTableSums 119b](#).

*< R\_ThreeTableSums 119b >* ≡

```
< R_ThreeTableSums Prototype 119a >
{
  SEXP ans, dim;
  < C integer N Input 23c >;
  < C integer Nsubset Input 26b >;
  int P, Q, B;

  N = XLENGTH(x);
  Nsubset = XLENGTH(subset);
  P = NLEVELS(x) + 1;
  Q = NLEVELS(y) + 1;
  B = NLEVELS(block);

  PROTECT(ans = allocVector(REALSXP, mPQB(P, Q, B)));
  PROTECT(dim = allocVector(INTSXP, 3));
  INTEGER(dim)[0] = P;
  INTEGER(dim)[1] = Q;
  INTEGER(dim)[2] = B;
  dimgets(ans, dim);
  RC_ThreeTableSums(INTEGER(x), N, P, INTEGER(y), Q,
                    INTEGER(block), B,
                    weights, subset, Offset0, Nsubset, REAL(ans));
  UNPROTECT(2);
  return(ans);
}
◇
```

Fragment referenced in [110a](#).

Defines: [R\\_ThreeTableSums 15b](#), [119a](#), [152c](#), [153](#).

Uses: [B 27a](#), [block 26f](#), [27b](#), [mPQB 132a](#), [N 23bc](#), [NLEVELS 131a](#), [Nsubset 26b](#), [Offset0 21b](#), [P 24a](#), [Q 24e](#), [RC\\_ThreeTableSums 120b](#), [subset 26ade](#), [weights 25b](#), [weights, 25cd](#), [x 23d](#), [24bc](#), [y 24df](#), [25a](#).

*< RC\_ThreeTableSums Prototype 120a >* ≡

```
void RC_ThreeTableSums
(
  < C ThreeTableSums Input 120c >
  < R weights Input 25b >,
  < R subset Input 26a >,
  < C subset range Input 26c >,
  < C ThreeTableSums Answer 121a >
)
```

Fragment referenced in [120b](#).  
Uses: [RC\\_ThreeTableSums 120b](#).

*< RC\_ThreeTableSums 120b >* ≡

```
< RC_ThreeTableSums Prototype 120a >
{
  if (TYPEOF(weights) == INTSXP) {
    if (TYPEOF(subset) == INTSXP) {
      C_ThreeTableSums_iweights_isubset(x, N, P, y, Q, block, B,
        INTEGER(weights), XLENGTH(weights) > 0, INTEGER(subset),
        offset, Nsubset, PQL_ans);
    } else {
      C_ThreeTableSums_iweights_dsubset(x, N, P, y, Q, block, B,
        INTEGER(weights), XLENGTH(weights) > 0, REAL(subset),
        offset, Nsubset, PQL_ans);
    }
  } else {
    if (TYPEOF(subset) == INTSXP) {
      C_ThreeTableSums_dweights_isubset(x, N, P, y, Q, block, B,
        REAL(weights), XLENGTH(weights) > 0, INTEGER(subset),
        offset, Nsubset, PQL_ans);
    } else {
      C_ThreeTableSums_dweights_dsubset(x, N, P, y, Q, block, B,
        REAL(weights), XLENGTH(weights) > 0, REAL(subset),
        offset, Nsubset, PQL_ans);
    }
  }
}
```

Fragment referenced in [110a](#).  
Defines: [RC\\_ThreeTableSums 41a](#), [119b](#), [120a](#).  
Uses: [B 27a](#), [block 26f](#), [27b](#), [C\\_ThreeTableSums\\_dweights\\_dsubset 121b](#), [C\\_ThreeTableSums\\_dweights\\_isubset 122a](#),  
[C\\_ThreeTableSums\\_iweights\\_dsubset 121c](#), [C\\_ThreeTableSums\\_iweights\\_isubset 121d](#), [N 23bc](#), [Nsubset 26b](#),  
[offset 26c](#), [P 24a](#), [Q 24e](#), [subset 26ade](#), [weights 25b](#), [x 23d](#), [24bc](#), [y 24df](#), [25a](#).

*< C ThreeTableSums Input 120c >* ≡

```
< C integer x Input 24c >
< C integer y Input 25a >
< C integer block Input 27b >
```

Fragment referenced in [120a](#), [121bcd](#), [122a](#).

*< C ThreeTableSums Answer 121a >* ≡

```
double *PQL_ans
◇
```

Fragment referenced in [120a](#), [121bcd](#), [122a](#).

*< C\_ThreeTableSums\_dweights\_dsubset 121b >* ≡

```
void C_ThreeTableSums_dweights_dsubset
(
  < C ThreeTableSums Input 120c >
  < C real weights Input 25d >
  < C real subset Input 26e >,
  < C ThreeTableSums Answer 121a >
) {
  double *s, *w;
  < ThreeTableSums Body 122b >
}
◇
```

Fragment referenced in [110a](#).

Defines: [C\\_ThreeTableSums\\_dweights\\_dsubset 120b](#).

*< C\_ThreeTableSums\_ieweights\_dsubset 121c >* ≡

```
void C_ThreeTableSums_ieweights_dsubset
(
  < C ThreeTableSums Input 120c >
  < C integer weights Input 25c >
  < C real subset Input 26e >,
  < C ThreeTableSums Answer 121a >
) {
  double *s;
  int *w;
  < ThreeTableSums Body 122b >
}
◇
```

Fragment referenced in [110a](#).

Defines: [C\\_ThreeTableSums\\_ieweights\\_dsubset 120b](#).

*< C\_ThreeTableSums\_ieweights\_isubset 121d >* ≡

```
void C_ThreeTableSums_ieweights_isubset
(
  < C ThreeTableSums Input 120c >
  < C integer weights Input 25c >
  < C integer subset Input 26d >,
  < C ThreeTableSums Answer 121a >
) {
  int *s, *w;
  < ThreeTableSums Body 122b >
}
◇
```

Fragment referenced in [110a](#).

Defines: [C\\_ThreeTableSums\\_ieweights\\_isubset 120b](#).

*< C\_ThreeTableSums\_dweights\_isubset 122a >* ≡

```
void C_ThreeTableSums_dweights_isubset
(
  < C ThreeTableSums Input 120c >
  < C real weights Input 25d >
  < C integer subset Input 26d >,
  < C ThreeTableSums Answer 121a >
) {
  int *s;
  double *w;
  < ThreeTableSums Body 122b >
}
◇
```

Fragment referenced in [110a](#).

Defines: `C_ThreeTableSums_dweights_isubset` [120b](#).

*< ThreeTableSums Body 122b >* ≡

```
int *xx, *yy, *bb, PQ = mPQB(P, Q, 1);

for (int p = 0; p < PQ * B; p++) PQL_ans[p] = 0.0;

yy = y;
xx = x;
bb = block;
< init subset loop 88b >
< start subset loop 88c >
{
  xx = xx + diff;
  yy = yy + diff;
  bb = bb + diff;
  if (HAS_WEIGHTS) {
    w = w + diff;
    PQL_ans[(bb[0] - 1) * PQ + yy[0] * P + xx[0]] += (double) w[0];
  } else {
    PQL_ans[(bb[0] - 1) * PQ + yy[0] * P + xx[0]]++;
  }
  < continue subset loop 89a >
}
xx = xx + diff;
yy = yy + diff;
bb = bb + diff;
if (HAS_WEIGHTS) {
  w = w + diff;
  PQL_ans[(bb[0] - 1) * PQ + yy[0] * P + xx[0]] += w[0];
} else {
  PQL_ans[(bb[0] - 1) * PQ + yy[0] * P + xx[0]]++;
}
}
◇
```

Fragment referenced in [121bcd](#), [122a](#).

Uses: `B` [27a](#), `block` [26f](#), [27b](#), `HAS_WEIGHTS` [25cd](#), `mPQB` [132a](#), `P` [24a](#), `Q` [24e](#), `x` [23d](#), [24bc](#), `y` [24df](#), [25a](#).

## 3.10 Utilities

### 3.10.1 Blocks

```
> sb <- sample(block)
> ns1 <- do.call(c, tapply(subset, sb[subset], function(i) i))
```

```
> ns2 <- .Call(libcoin:::R_order_subset_wrt_block, y, integer(0), subset, sb)
> stopifnot(isequal(ns1, ns2))
```

*< Utils 123a >* ≡

```
  < C_setup_subset 125a >
  < C_setup_subset_block 125b >
  < C_order_subset_wrt_block 126a >
  < RC_order_subset_wrt_block 124b >
  < R_order_subset_wrt_block 123c >
  ◇
```

Fragment referenced in [23a](#).

*< R\_order\_subset\_wrt\_block Prototype 123b >* ≡

```
SEXP R_order_subset_wrt_block
(
  < R y Input 24d >
  < R weights Input 25b >,
  < R subset Input 26a >,
  < R block Input 26f >
)
◇
```

Fragment referenced in [22b](#), [123c](#).

Uses: [R\\_order\\_subset\\_wrt\\_block 123c](#).

*< R\_order\_subset\_wrt\_block 123c >* ≡

```
< R_order_subset_wrt_block Prototype 123b >
{
  < C integer N Input 23c >;
  SEXP blockTable, ans;

  N = XLENGTH(y) / NCOL(y);

  if (XLENGTH(weights) > 0)
    error("cannot deal with weights here");

  if (NLEVELS(block) > 1) {
    PROTECT(blockTable = R_OneTableSums(block, weights, subset));
  } else {
    PROTECT(blockTable = allocVector(REALSXP, 2));
    REAL(blockTable)[0] = 0.0;
    REAL(blockTable)[1] = RC_Sums(N, weights, subset, Offset0, XLENGTH(subset));
  }

  PROTECT(ans = RC_order_subset_wrt_block(N, subset, block, blockTable));

  UNPROTECT(2);
  return(ans);
}
◇
```

Fragment referenced in [123a](#).

Defines: [R\\_order\\_subset\\_wrt\\_block 123b](#), [152c](#), [153](#).

Uses: [block 26f](#), [27b](#), [blockTable 27c](#), [N 23bc](#), [NCOL 130c](#), [NLEVELS 131a](#), [Offset0 21b](#), [RC\\_order\\_subset\\_wrt\\_block 124b](#), [RC\\_Sums 91a](#), [R\\_OneTableSums 111a](#), [subset 26ade](#), [weights 25b](#), [weights, 25cd](#), [y 24df](#), [25a](#).

$\langle RC\_order\_subset\_wrt\_block \text{ Prototype 124a} \rangle \equiv$

```
SEXP RC_order_subset_wrt_block
(
   $\langle C \text{ integer } N \text{ Input 23c} \rangle$ ,
   $\langle R \text{ subset Input 26a} \rangle$ ,
   $\langle R \text{ block Input 26f} \rangle$ ,
   $\langle R \text{ blockTable Input 27c} \rangle$ 
)
◇
```

Fragment referenced in [124b](#).

Uses: [RC\\_order\\_subset\\_wrt\\_block 124b](#).

$\langle RC\_order\_subset\_wrt\_block \text{ 124b} \rangle \equiv$

```
 $\langle RC\_order\_subset\_wrt\_block \text{ Prototype 124a} \rangle$ 
{
  SEXP ans;
  int NOBLOCK = (XLENGTH(block) == 0 || XLENGTH(blockTable) == 2);

  if (XLENGTH(subset) > 0) {
    if (NOBLOCK) {
      return(subset);
    } else {
      PROTECT(ans = allocVector(TYPEOF(subset), XLENGTH(subset)));
      C_order_subset_wrt_block(subset, block, blockTable, ans);
      UNPROTECT(1);
      return(ans);
    }
  } else {
    PROTECT(ans = allocVector(TYPEOF(subset), N));
    if (NOBLOCK) {
      C_setup_subset(N, ans);
    } else {
      C_setup_subset_block(N, block, blockTable, ans);
    }
    UNPROTECT(1);
    return(ans);
  }
}
◇
```

Fragment referenced in [123a](#).

Defines: [RC\\_order\\_subset\\_wrt\\_block 34a](#), [37](#), [123c](#), [124a](#).

Uses: [block 26f](#), [27b](#), [blockTable 27c](#), [C\\_order\\_subset\\_wrt\\_block 126a](#), [C\\_setup\\_subset 125a](#),  
[C\\_setup\\_subset\\_block 125b](#), [N 23bc](#), [subset 26ade](#).

`< C_setup_subset 125a > ≡`

```
void C_setup_subset
(
  < C integer N Input 23c >,
  SEXP ans
) {
  for (R_xlen_t i = 0; i < N; i++) {
    /* ans is R style index in 1:N */
    if (TYPEOF(ans) == INTSXP) {
      INTEGER(ans)[i] = i + 1;
    } else {
      REAL(ans)[i] = (double) i + 1;
    }
  }
}
◇
```

Fragment referenced in [123a](#).  
Defines: `C_setup_subset` [124b](#), [127a](#).  
Uses: `N` [23bc](#).

`< C_setup_subset_block 125b > ≡`

```
void C_setup_subset_block
(
  < C integer N Input 23c >,
  < R block Input 26f >,
  < R blockTable Input 27c >,
  SEXP ans
) {
  double *cumtable;
  int Nlevels = LENGTH(blockTable);

  cumtable = R_Calloc(Nlevels, double);
  for (int k = 0; k < Nlevels; k++) cumtable[k] = 0.0;

  /* table[0] are missings, ie block == 0 ! */
  for (int k = 1; k < Nlevels; k++)
    cumtable[k] = cumtable[k - 1] + REAL(blockTable)[k - 1];

  for (R_xlen_t i = 0; i < N; i++) {
    /* ans is R style index in 1:N */
    if (TYPEOF(ans) == INTSXP) {
      INTEGER(ans)[(int) cumtable[INTEGER(block)[i]]++] = i + 1;
    } else {
      REAL(ans)[(R_xlen_t) cumtable[INTEGER(block)[i]]++] = (double) i + 1;
    }
  }

  R_Free(cumtable);
}
◇
```

Fragment referenced in [123a](#).  
Defines: `C_setup_subset_block` [124b](#).  
Uses: `block` [26f](#), [27b](#), `blockTable` [27c](#), `N` [23bc](#).

$\langle C\_order\_subset\_wrt\_block\ 126a \rangle \equiv$

```
void C_order_subset_wrt_block
(
   $\langle R\ subset\ Input\ 26a \rangle$ ,
   $\langle R\ block\ Input\ 26f \rangle$ ,
   $\langle R\ blockTable\ Input\ 27c \rangle$ ,
  SEXP ans
) {
  double *cumtable;
  int Nlevels = LENGTH(blockTable);

  cumtable = R_Calloc(Nlevels, double);
  for (int k = 0; k < Nlevels; k++) cumtable[k] = 0.0;

  /* table[0] are missings, ie block == 0 ! */
  for (int k = 1; k < Nlevels; k++)
    cumtable[k] = cumtable[k - 1] + REAL(blockTable)[k - 1];

  /* subset is R style index in 1:N */
  if (typeof(subset) == INTSXP) {
    for (R_xlen_t i = 0; i < XLENGTH(subset); i++)
      INTEGER(ans)[(int) cumtable[INTEGER(block)[INTEGER(subset)[i] - 1]]++] = INTEGER(subset)[i];
  } else {
    for (R_xlen_t i = 0; i < XLENGTH(subset); i++)
      REAL(ans)[(R_xlen_t) cumtable[INTEGER(block)[(R_xlen_t) REAL(subset)[i] - 1]]++] = REAL(subset)[i];
  }

  R_Free(cumtable);
}
◇
```

Fragment referenced in [123a](#).

Defines: [C\\_order\\_subset\\_wrt\\_block 124b](#).

Uses: [block 26f](#), [27b](#), [blockTable 27c](#), [N 23bc](#), [subset 26ade](#).

$\langle RC\_setup\_subset\ Prototype\ 126b \rangle \equiv$

```
SEXP RC_setup_subset
(
   $\langle C\ integer\ N\ Input\ 23c \rangle$ ,
   $\langle R\ weights\ Input\ 25b \rangle$ ,
   $\langle R\ subset\ Input\ 26a \rangle$ 
)
◇
```

Fragment referenced in [127a](#).

Uses: [RC\\_setup\\_subset 127a](#).

Because this will only be used when really needed (in Permutations) we can be a little bit more generous with memory here. The return value is always REALSXP.

*< RC\_setup\_subset 127a >* ≡

*< RC\_setup\_subset Prototype 126b >*

```
{
  SEXP ans, mysubset;
  R_xlen_t sumweights;

  if (XLENGTH(subset) == 0) {
    PROTECT(mysubset = allocVector(REALSXP, N));
    C_setup_subset(N, mysubset);
  } else {
    PROTECT(mysubset = coerceVector(subset, REALSXP));
  }

  if (XLENGTH(weights) == 0) {
    UNPROTECT(1);
    return(mysubset);
  }

  sumweights = (R_xlen_t) RC_Sums(N, weights, mysubset, Offset0, XLENGTH(subset));
  PROTECT(ans = allocVector(REALSXP, sumweights));

  R_xlen_t itmp = 0;
  for (R_xlen_t i = 0; i < XLENGTH(mysubset); i++) {
    if (TYPEOF(weights) == REALSXP) {
      for (R_xlen_t j = 0; j < REAL(weights)[(R_xlen_t) REAL(mysubset)[i] - 1]; j++)
        REAL(ans)[itmp++] = REAL(mysubset)[i];
    } else {
      for (R_xlen_t j = 0; j < INTEGER(weights)[(R_xlen_t) REAL(mysubset)[i] - 1]; j++)
        REAL(ans)[itmp++] = REAL(mysubset)[i];
    }
  }
  UNPROTECT(2);
  return(ans);
}
◇
```

Fragment referenced in [127b](#).

Defines: [RC\\_setup\\_subset 37](#), [126b](#).

Uses: [C\\_setup\\_subset 125a](#), [N 23bc](#), [Offset0 21b](#), [RC\\_Sums 91a](#), [subset 26ade](#), [sumweights 25e](#), [weights 25b](#), [weights, 25cd](#).

### 3.10.2 Permutation Helpers

*< Permutations 127b >* ≡

```
< RC_setup_subset 127a >
< C_Permute 128a >
< C_doPermute 128b >
< C_PermuteBlock 129a >
< C_doPermuteBlock 129b >
◇
```

Fragment referenced in [23a](#).

$\langle C\_Permute\ 128a \rangle \equiv$

```
void C_Permute
(
    double *subset,
     $\langle C\ integer\ Nsubset\ Input\ 26b \rangle$ ,
    double *ans
) {
    R_xlen_t n = Nsubset, j;

    for (R_xlen_t i = 0; i < Nsubset; i++) {
        j = n * unif_rand();
        ans[i] = subset[j];
        subset[j] = subset[--n];
    }
}
◇
```

Fragment referenced in [127b](#).  
Defines: [C\\_Permute 128b](#), [129a](#).  
Uses: [Nsubset 26b](#), [subset 26ade](#).

$\langle C\_doPermute\ 128b \rangle \equiv$

```
void C_doPermute
(
    double *subset,
     $\langle C\ integer\ Nsubset\ Input\ 26b \rangle$ ,
    double *Nsubset_tmp,
    double *perm
) {
    Memcpy(Nsubset_tmp, subset, Nsubset);
    C_Permute(Nsubset_tmp, Nsubset, perm);
}
◇
```

Fragment referenced in [127b](#).  
Defines: [C\\_doPermute 37](#).  
Uses: [C\\_Permute 128a](#), [Nsubset 26b](#), [subset 26ade](#).

$\langle C\_PermuteBlock\ 129a \rangle \equiv$

```
void C_PermuteBlock
(
    double *subset,
    double *table,
    int Nlevels,
    double *ans
) {
    double *px, *pans;

    px = subset;
    pans = ans;

    for (R_xlen_t j = 0; j < Nlevels; j++) {
        if (table[j] > 0) {
            C_Permute(px, (R_xlen_t) table[j], pans);
            px += (R_xlen_t) table[j];
            pans += (R_xlen_t) table[j];
        }
    }
}
◇
```

Fragment referenced in [127b](#).

Defines: [C\\_PermuteBlock 129b](#).

Uses: [C\\_Permute 128a](#), [subset 26ade](#).

$\langle C\_doPermuteBlock\ 129b \rangle \equiv$

```
void C_doPermuteBlock
(
    double *subset,
     $\langle C\ integer\ Nsubset\ Input\ 26b \rangle$ ,
    double *table,
    int Nlevels,
    double *Nsubset_tmp,
    double *perm
) {
    Memcpy(Nsubset_tmp, subset, Nsubset);
    C_PermuteBlock(Nsubset_tmp, table, Nlevels, perm);
}
◇
```

Fragment referenced in [127b](#).

Defines: [C\\_doPermuteBlock 37](#).

Uses: [C\\_PermuteBlock 129a](#), [Nsubset 26b](#), [subset 26ade](#).

### 3.10.3 Other Utils

`< MoreUtils 130a > ≡`

```
< NROW 130b >
< NCOL 130c >
< NLEVELS 131a >
< C_kronecker 133b >
< R_kronecker 133a >
< C_kronecker_sym 134 >
< C_KronSums_sym 135a >
< C_MPinv_sym 137 >
< R_MPinv_sym 136b >
< R_unpack_sym 139 >
< R_pack_sym 140c >
◇
```

Fragment referenced in [23a](#).

`< NROW 130b > ≡`

```
int NROW
(
    SEXP x
) {
    SEXP a;
    a = getAttrib(x, R_DimSymbol);
    if (a == R_NilValue) return(XLENGTH(x));
    if (TYPEOF(a) == REALSXP)
        return(REAL(a)[0]);
    return(INTEGER(a)[0]);
}
◇
```

Fragment referenced in [130a](#).

Defines: [NROW 6, 8, 9ab, 14, 33a, 37, 43b, 44, 61b, 131a, 133a, 140c](#).

Uses: [x 23d, 24bc](#).

`< NCOL 130c > ≡`

```
int NCOL
(
    SEXP x
) {
    SEXP a;
    a = getAttrib(x, R_DimSymbol);
    if (a == R_NilValue) return(1);
    if (TYPEOF(a) == REALSXP)
        return(REAL(a)[1]);
    return(INTEGER(a)[1]);
}
◇
```

Fragment referenced in [130a](#).

Defines: [NCOL 12, 31b, 41b, 61b, 81a, 83a, 94b, 102b, 106c, 123c, 133a](#).

Uses: [x 23d, 24bc](#).

$\langle NLEVELS\ 131a \rangle \equiv$

```
int NLEVELS
(
  SEXP x
) {
  SEXP a;
  int maxlev = 0;

  a = getAttrib(x, R_LevelsSymbol);
  if (a == R_NilValue) {
    if (TYPEOF(x) != INTSXP)
      error("cannot determine number of levels");
    for (R_xlen_t i = 0; i < XLENGTH(x); i++) {
      if (INTEGER(x)[i] > maxlev)
        maxlev = INTEGER(x)[i];
    }
    return(maxlev);
  }
  return(NROW(a));
}
◇
```

Fragment referenced in 130a.

Defines: NLEVELS 31b, 41b, 111a, 115a, 119b, 123c.

Uses: NROW 130b, x 23d, 24bc.

Check for integer overflow when computing  $P(P + 1)/2$  and  $PQ$ .

$\langle PP12\ 131b \rangle \equiv$

```
int PP12
(
  int P
) {
  double dP = (double) P;
  double ans;

  ans = dP * (dP + 1) / 2;

  if (ans > INT_MAX)
    error("cannot allocate memory: number of levels too large");

  return((int) ans);
}
◇
```

Fragment referenced in 141a.

Defines: PP12 34a, 44, 46, 51, 79, 88a, 148, 149a.

Uses: P 24a.

$\langle$  *mPQB* 132a  $\rangle \equiv$

```
int mPQB
(
  int P,
  int Q,
  int B
) {
  double ans = P * Q * B;

  if (ans > INT_MAX)
    error("cannot allocate memory: number of levels too large");

  return((int) ans);
}
◇
```

Fragment referenced in [141a](#).

Defines: *mPQB* [36a](#), [37](#), [45](#), [48](#), [52a](#), [71](#), [73a](#), [76b](#), [78b](#), [79](#), [80a](#), [101b](#), [105c](#), [115a](#), [119b](#), [122b](#), [148](#).

Uses: *B* [27a](#), *P* [24a](#), *Q* [24e](#).

```
> A <- matrix(runif(12), ncol = 3)
> B <- matrix(runif(10), ncol = 2)
> K1 <- kronecker(A, B)
> K2 <- .Call(libcoin::R_kronecker, A, B)
> stopifnot(isequal(K1, K2))
```

$\langle$  *R\_kronecker Prototype* 132b  $\rangle \equiv$

```
SEXP R_kronecker
(
  SEXP A,
  SEXP B
)
◇
```

Fragment referenced in [22b](#), [133a](#).

Uses: *B* [27a](#).

This function can be called from other packages.

"libcoinAPI.h" 132c  $\equiv$

```
extern SEXP libcoin_R_kronecker(
  SEXP A, SEXP B
) {
  static SEXP(*fun)(SEXP, SEXP) = NULL;
  if (fun == NULL)
    fun = (SEXP*)(SEXP, SEXP)
      R_GetCCallable("libcoin", "R_kronecker");
  return fun(A, B);
}
◇
```

File defined by [30d](#), [36c](#), [38c](#), [40b](#), [47b](#), [50c](#), [53](#), [55b](#), [61a](#), [132c](#), [136a](#), [138b](#), [140b](#).

Uses: *B* [27a](#).

$\langle R\_kronecker\ 133a \rangle \equiv$

$\langle R\_kronecker\ Prototype\ 132b \rangle$

```
{
    int m, n, r, s;
    SEXP ans;

    if (!isReal(A) || !isReal(B))
        error("R_kronecker: A and / or B are not of type REALSXP");

    m = NROW(A);
    n = NCOL(A);
    r = NROW(B);
    s = NCOL(B);

    PROTECT(ans = allocMatrix(REALSXP, m * n, r * s));
    C_kronecker(REAL(A), m, n, REAL(B), r, s, 1, REAL(ans));
    UNPROTECT(1);
    return(ans);
}
◇
```

Fragment referenced in [130a](#).

Uses: [B 27a](#), [C\\_kronecker 133b](#), [NCOL 130c](#), [NROW 130b](#).

$\langle C\_kronecker\ 133b \rangle \equiv$

```
void C_kronecker
(
    const double *A,
    const int m,
    const int n,
    const double *B,
    const int r,
    const int s,
    const int overwrite,
    double *ans
) {
    int mr, js, ir;
    double y;

    if (overwrite) {
        for (int i = 0; i < m * r * n * s; i++) ans[i] = 0.0;
    }

    mr = m * r;
    for (int i = 0; i < m; i++) {
        ir = i * r;
        for (int j = 0; j < n; j++) {
            js = j * s;
            y = A[j*m + i];
            for (int k = 0; k < r; k++) {
                for (int l = 0; l < s; l++)
                    ans[(js + l) * mr + ir + k] += y * B[l * r + k];
            }
        }
    }
}
◇
```

Fragment referenced in [130a](#).

Defines: [C\\_kronecker 80a](#), [133a](#).

Uses: [B 27a](#), [y 24df](#), [25a](#).

$\langle C\_kronecker\_sym\ 134 \rangle \equiv$

```
void C_kronecker_sym
(
    const double *A,
    const int m,
    const double *B,
    const int r,
    const int overwrite,
    double *ans
) {
    int mr, js, ir, s;
    double y;

    mr = m * r;
    s = r;

    if (overwrite) {
        for (int i = 0; i < mr * (mr + 1) / 2; i++) ans[i] = 0.0;
    }

    for (int i = 0; i < m; i++) {
        ir = i * r;
        for (int j = 0; j <= i; j++) {
            js = j * s;
            y = A[S(i, j, m)];
            for (int k = 0; k < r; k++) {
                for (int l = 0; l < (j < i ? s : k + 1); l++) {
                    ans[S(ir + k, js + l, mr)] += y * B[S(k, l, r)];
                }
            }
        }
    }
}
◇
```

Fragment referenced in 130a.  
Defines: C\_kronecker\_sym 79.  
Uses: B 27a, S 21a, y 24df, 25a.

$\langle C\_KronSums\_sym\ 135a \rangle \equiv$

```
/* sum_i (t(x[i,]) %% x[i,]) */
void C_KronSums_sym_
(
   $\langle C\ real\ x\ Input\ 24b \rangle$ 
  double *PP_sym_ans
) {
  int pN, qN, SpqP;

  for (int q = 0; q < P; q++) {
    qN = q * N;
    for (int p = 0; p <= q; p++) {
      PP_sym_ans[S(p, q, P)] = 0.0;
      pN = p * N;
      SpqP = S(p, q, P);
      for (int i = 0; i < N; i++)
        PP_sym_ans[SpqP] += x[qN + i] * x[pN + i];
    }
  }
}
◇
```

Fragment referenced in [130a](#).

Defines: C\_KronSums\_sym Never used.

Uses: N [23bc](#), P [24a](#), S [21a](#), x [23d](#), [24bc](#).

```
> covar <- vcov(ls1)
> covar_sym <- ls1$Covariance
> n <- (sqrt(1 + 8 * length(covar_sym)) - 1) / 2
> tol <- sqrt(.Machine$double.eps)
> MP1 <- MPinverse(covar, tol)
> MP2 <- .Call(libcoin:::R_MPinv_sym, covar_sym, as.integer(n), tol)
> lt <- lower.tri(covar, diag = TRUE)
> stopifnot(isequal(MP1$MPinv[lt], MP2$MPinv) &&
+           isequal(MP1$rank, MP2$rank))
```

$\langle R\_MPinv\_sym\ Prototype\ 135b \rangle \equiv$

```
SEXP R_MPinv_sym
(
  SEXP x,
  SEXP n,
  SEXP tol
)
◇
```

Fragment referenced in [22b](#), [136b](#).

Uses: R\_MPinv\_sym [136b](#), x [23d](#), [24bc](#).

This function can be called from other packages.

"libcoinAPI.h" 136a≡

```
extern SEXP libcoin_R_MPinv_sym(  
  SEXP x, SEXP n, SEXP tol  
) {  
  static SEXP(*fun)(SEXP, SEXP, SEXP) = NULL;  
  if (fun == NULL)  
    fun = (SEXP*)(SEXP, SEXP, SEXP)  
      R_GetCCallable("libcoin", "R_MPinv_sym");  
  return fun(x, n, tol);  
}  
◇
```

File defined by [30d](#), [36c](#), [38c](#), [40b](#), [47b](#), [50c](#), [53](#), [55b](#), [61a](#), [132c](#), [136a](#), [138b](#), [140b](#).  
Uses: [R\\_MPinv\\_sym 136b](#), [x 23d](#), [24bc](#).

⟨ *R\_MPinv\_sym 136b* ⟩ ≡

```
⟨ R_MPinv_sym Prototype 135b ⟩  
{  
  int m;  
  SEXP ans, names, MPinv, rank;  
  
  m = INTEGER(n)[0];  
  if (m == 0)  
    m = (int) (sqrt(0.25 + 2 * LENGTH(x)) - 0.5);  
  
  PROTECT(ans = allocVector(VECSXP, 2));  
  PROTECT(names = allocVector(STRSXP, 2));  
  SET_VECTOR_ELT(ans, 0, MPinv = allocVector(REALSXP, LENGTH(x)));  
  SET_STRING_ELT(names, 0, mkChar("MPinv"));  
  SET_VECTOR_ELT(ans, 1, rank = allocVector(INTSXP, 1));  
  SET_STRING_ELT(names, 1, mkChar("rank"));  
  namesgets(ans, names);  
  
  C_MPinv_sym(REAL(x), m, REAL(tol)[0], REAL(MPinv), INTEGER(rank));  
  
  UNPROTECT(2);  
  return(ans);  
}  
◇
```

Fragment referenced in [130a](#).  
Defines: [R\\_MPinv\\_sym 135b](#), [136a](#), [152c](#), [153](#).  
Uses: [x 23d](#), [24bc](#).

$\langle C\_MPinv\_sym\ 137 \rangle \equiv$

```
void C_MPinv_sym
(
    const double *x,
    const int n,
    const double tol,
    double *dMP,
    int *rank
) {
    double *val, *vec, dtol, *rx, *work, valinv;
    int valzero = 0, info = 0, kn;

    if (n == 1) {
        if (x[0] > tol) {
            dMP[0] = 1 / x[0];
            rank[0] = 1;
        } else {
            dMP[0] = 0;
            rank[0] = 0;
        }
    } else {
        rx = R_Calloc(n * (n + 1) / 2, double);
        Memcpy(rx, x, n * (n + 1) / 2);
        work = R_Calloc(3 * n, double);
        val = R_Calloc(n, double);
        vec = R_Calloc(n * n, double);

        F77_CALL(dspev)("V", "L", &n, rx, val, vec, &n, work,
            &info FCONE FCONE);

        dtol = val[n - 1] * tol;

        for (int k = 0; k < n; k++)
            valzero += (val[k] < dtol);
        rank[0] = n - valzero;

        for (int k = 0; k < n * (n + 1) / 2; k++) dMP[k] = 0.0;

        for (int k = valzero; k < n; k++) {
            valinv = 1 / val[k];
            kn = k * n;
            for (int i = 0; i < n; i++) {
                for (int j = 0; j <= i; j++) {
                    /* MP is symmetric */
                    dMP[S(i, j, n)] += valinv * vec[kn + i] * vec[kn + j];
                }
            }
        }
        R_Free(rx); R_Free(work); R_Free(val); R_Free(vec);
    }
}
◇
```

Fragment referenced in 130a.

Uses: S 21a, x 23d, 24bc.

```
> m <- matrix(c(3, 2, 1,
+             2, 4, 2,
+             1, 2, 5),
+            ncol = 3)
> s <- m[lower.tri(m, diag = TRUE)]
> u1 <- .Call(libcoin:::R_unpack_sym, s, NULL, 0L)
```

```
> u2 <- .Call(libcoin:::R_unpack_sym, s, NULL, 1L)
> stopifnot(isequal(m, u1) && isequal(diag(m), u2))
```

*<R\_unpack\_sym Prototype 138a> ≡*

```
SEXP R_unpack_sym
(
  SEXP x,
  SEXP names,
  SEXP diagonly
)
◇
```

Fragment referenced in [22b](#), [139](#).

Uses: [R\\_unpack\\_sym 139](#), [x 23d](#), [24bc](#).

This function can be called from other packages.

*"libcoinAPI.h" 138b≡*

```
extern SEXP libcoin_R_unpack_sym(
  SEXP x, SEXP names, SEXP diagonly
) {
  static SEXP(*fun)(SEXP, SEXP, SEXP) = NULL;
  if (fun == NULL)
    fun = (SEXP*)(SEXP, SEXP, SEXP)
      R_GetCCallable("libcoin", "R_unpack_sym");
  return fun(x, names, diagonly);
}
◇
```

File defined by [30d](#), [36c](#), [38c](#), [40b](#), [47b](#), [50c](#), [53](#), [55b](#), [61a](#), [132c](#), [136a](#), [138b](#), [140b](#).

Uses: [R\\_unpack\\_sym 139](#), [x 23d](#), [24bc](#).

$\langle R\_unpack\_sym\ 139 \rangle \equiv$

$\langle R\_unpack\_sym\ Prototype\ 138a \rangle$

```
{
  R_xlen_t n, k = 0;
  SEXP ans, dimnames;
  double *dx, *dans;

  /* m = n * (n + 1)/2 <=> n^2 + n - 2 * m = 0 */
  n = sqrt(0.25 + 2 * XLENGTH(x)) - 0.5;

  dx = REAL(x);
  if (INTEGER(diagonly)[0]) {
    PROTECT(ans = allocVector(REALSXP, n));
    if (names != R_NilValue) {
      namesgets(ans, names);
    }
    dans = REAL(ans);
    for (R_xlen_t i = 0; i < n; i++) {
      dans[i] = dx[k];
      k += n - i;
    }
  } else {
    PROTECT(ans = allocMatrix(REALSXP, n, n));
    if (names != R_NilValue) {
      PROTECT(dimnames = allocVector(VECSXP, 2));
      SET_VECTOR_ELT(dimnames, 0, names);
      SET_VECTOR_ELT(dimnames, 1, names);
      dimnamesgets(ans, dimnames);
      UNPROTECT(1);
    }
    dans = REAL(ans);
    for (R_xlen_t i = 0; i < n; i++) {
      dans[i * n + i] = dx[k]; /* diagonal */
      k++;
      for (R_xlen_t j = i + 1; j < n; j++) {
        dans[i * n + j] = dx[k]; /* lower triangular */
        dans[j * n + i] = dx[k]; /* upper triangular */
        k++;
      }
    }
  }
  UNPROTECT(1);
  return ans;
}
◇
```

Fragment referenced in 130a.

Defines: `R_unpack_sym` 10, 138ab, 152c, 153.

Uses: x 23d, 24bc.

```
> m <- matrix(c(4, 3, 2, 1,
+             3, 5, 4, 2,
+             2, 4, 6, 5,
+             1, 2, 5, 7),
+           ncol = 4)
> s <- m[lower.tri(m, diag = TRUE)]
> p <- .Call(libcoin:::R_pack_sym, m)
> stopifnot(isequal(s, p))
```

*< R\_pack\_sym Prototype 140a >* ≡

```
SEXP R_pack_sym
(
    SEXP x
)
◇
```

Fragment referenced in 22b, 140c.

Uses: R\_pack\_sym 140c, x 23d, 24bc.

This function can be called from other packages.

"libcoinAPI.h" 140b≡

```
extern SEXP libcoin_R_pack_sym(
    SEXP x
) {
    static SEXP(*fun)(SEXP) = NULL;
    if (fun == NULL)
        fun = (SEXP*)(SEXP)
            R_GetCCallable("libcoin", "R_pack_sym");
    return fun(x);
}
◇
```

File defined by 30d, 36c, 38c, 40b, 47b, 50c, 53, 55b, 61a, 132c, 136a, 138b, 140b.

Uses: R\_pack\_sym 140c, x 23d, 24bc.

*< R\_pack\_sym 140c >* ≡

```
< R_pack_sym Prototype 140a >
{
    R_xlen_t n, k = 0;
    SEXP ans;
    double *dx, *dans;

    n = NROW(x);
    dx = REAL(x);
    PROTECT(ans = allocVector(REALSXP, n * (n + 1) / 2));
    dans = REAL(ans);

    for (R_xlen_t i = 0; i < n; i++) {
        for (R_xlen_t j = i; j < n; j++) {
            dans[k] = dx[i * n + j];
            k++;
        }
    }

    UNPROTECT(1);
    return ans;
}
◇
```

Fragment referenced in 130a.

Defines: R\_pack\_sym 140ab, 152c, 153.

Uses: NROW 130b, x 23d, 24bc.

## 3.11 Memory

$\langle$  *Memory* 141a  $\rangle \equiv$

$\langle$  *C\_get\_P* 141c  $\rangle$   
 $\langle$  *C\_get\_Q* 142a  $\rangle$   
 $\langle$  *PP12* 131b  $\rangle$   
 $\langle$  *mPQB* 132a  $\rangle$   
 $\langle$  *C\_get\_varonly* 142b  $\rangle$   
 $\langle$  *C\_get\_Xfactor* 142c  $\rangle$   
 $\langle$  *C\_get\_LinearStatistic* 142d  $\rangle$   
 $\langle$  *C\_get\_Expectation* 143a  $\rangle$   
 $\langle$  *C\_get\_Variance* 143b  $\rangle$   
 $\langle$  *C\_get\_Covariance* 143c  $\rangle$   
 $\langle$  *C\_get\_ExpectationX* 144a  $\rangle$   
 $\langle$  *C\_get\_ExpectationInfluence* 144b  $\rangle$   
 $\langle$  *C\_get\_CovarianceInfluence* 144c  $\rangle$   
 $\langle$  *C\_get\_VarianceInfluence* 144d  $\rangle$   
 $\langle$  *C\_get\_TableBlock* 145a  $\rangle$   
 $\langle$  *C\_get\_Sumweights* 145b  $\rangle$   
 $\langle$  *C\_get\_Table* 145c  $\rangle$   
 $\langle$  *C\_get\_dimTable* 145d  $\rangle$   
 $\langle$  *C\_get\_B* 146a  $\rangle$   
 $\langle$  *C\_get\_nresample* 146b  $\rangle$   
 $\langle$  *C\_get\_PermutedLinearStatistic* 146c  $\rangle$   
 $\langle$  *C\_get\_tol* 146d  $\rangle$   
 $\langle$  *RC\_init\_LECV\_1d* 149b  $\rangle$   
 $\langle$  *RC\_init\_LECV\_2d* 150  $\rangle$   
 $\diamond$

Fragment referenced in 23a.

$\langle$  *R\_LECV\_Input* 141b  $\rangle \equiv$

SEXP LECV  
 $\diamond$

Fragment referenced in 50b, 52b, 141c, 142abcd, 143abc, 144abcd, 145abcd, 146abcd.

Defines: LECV 38bc, 39a, 50c, 51, 52a, 53, 54, 55ab, 56, 69b, 71, 141c, 142abcd, 143abc, 144abcd, 145abcd, 146abcd.

$\langle$  *C\_get\_P* 141c  $\rangle \equiv$

```
int C_get_P
(
     $\langle$  R_LECV_Input 141b  $\rangle$ 
) {
    return(INTEGER(VECTOR_ELT(LECV, dim_SLOT))[0]);
}
 $\diamond$ 
```

Fragment referenced in 141a.

Defines: C\_get\_P 33a, 39a, 46, 52a, 56, 71, 143bc, 146b.

Uses: dim\_SLOT 21b, LECV 141b.

$\langle C\_get\_Q\ 142a \rangle \equiv$

```
int C_get_Q
(
     $\langle R\ LECV\ Input\ 141b \rangle$ 
) {
    return(INTEGER(VECTOR_ELT(LECV, dim_SLOT))[1]);
}
◇
```

Fragment referenced in 141a.

Defines: C\_get\_Q 33a, 39a, 46, 52a, 71, 143bc, 146b.

Uses: dim\_SLOT 21b, LECV 141b.

$\langle C\_get\_varonly\ 142b \rangle \equiv$

```
int C_get_varonly
(
     $\langle R\ LECV\ Input\ 141b \rangle$ 
) {
    return(INTEGER(VECTOR_ELT(LECV, varonly_SLOT))[0]);
}
◇
```

Fragment referenced in 141a.

Defines: C\_get\_varonly 32, 34a, 36a, 39a, 44, 45, 46, 52a, 54, 71, 143c.

Uses: LECV 141b, varonly\_SLOT 21b.

$\langle C\_get\_Xfactor\ 142c \rangle \equiv$

```
int C_get_Xfactor
(
     $\langle R\ LECV\ Input\ 141b \rangle$ 
) {
    return(INTEGER(VECTOR_ELT(LECV, Xfactor_SLOT))[0]);
}
◇
```

Fragment referenced in 141a.

Defines: C\_get\_Xfactor 46.

Uses: LECV 141b, Xfactor\_SLOT 21b.

$\langle C\_get\_LinearStatistic\ 142d \rangle \equiv$

```
double* C_get_LinearStatistic
(
     $\langle R\ LECV\ Input\ 141b \rangle$ 
) {
    return(REAL(VECTOR_ELT(LECV, LinearStatistic_SLOT)));
}
◇
```

Fragment referenced in 141a.

Defines: C\_get\_LinearStatistic 33b, 45, 51, 54, 71, 149a.

Uses: LECV 141b, LinearStatistic\_SLOT 21b.

*< C\_get\_Expectation 143a >* ≡

```
double* C_get_Expectation
(
    < R LECV Input 141b >
) {
    return(REAL(VECTOR_ELT(LECV, Expectation_SLOT)));
}
◇
```

Fragment referenced in [141a](#).

Defines: [C\\_get\\_Expectation 35a, 39a, 43b, 51, 54, 71, 149a](#).

Uses: [Expectation\\_SLOT 21b, LECV 141b](#).

*< C\_get\_Variance 143b >* ≡

```
double* C_get_Variance
(
    < R LECV Input 141b >
) {
    int PQ = C_get_P(LECV) * C_get_Q(LECV);
    double *var, *covar;

    if (isNull(VECTOR_ELT(LECV, Variance_SLOT))) {
        SET_VECTOR_ELT(LECV, Variance_SLOT,
                       allocVector(REALSXP, PQ));
        if (!isNull(VECTOR_ELT(LECV, Covariance_SLOT))) {
            covar = REAL(VECTOR_ELT(LECV, Covariance_SLOT));
            var = REAL(VECTOR_ELT(LECV, Variance_SLOT));
            for (int p = 0; p < PQ; p++)
                var[p] = covar[S(p, p, PQ)];
        }
    }
    return(REAL(VECTOR_ELT(LECV, Variance_SLOT)));
}
◇
```

Fragment referenced in [141a](#).

Defines: [C\\_get\\_Variance 35c, 36a, 39a, 44, 45, 54, 71, 143c, 149a](#).

Uses: [Covariance\\_SLOT 21b, C\\_get\\_P 141c, C\\_get\\_Q 142a, LECV 141b, S 21a, Variance\\_SLOT 21b](#).

*< C\_get\_Covariance 143c >* ≡

```
double* C_get_Covariance
(
    < R LECV Input 141b >
) {
    int PQ = C_get_P(LECV) * C_get_Q(LECV);
    if (C_get_varonly(LECV) && PQ > 1)
        error("Cannot extract covariance from variance only object");
    if (C_get_varonly(LECV) && PQ == 1)
        return(C_get_Variance(LECV));
    return(REAL(VECTOR_ELT(LECV, Covariance_SLOT)));
}
◇
```

Fragment referenced in [141a](#).

Defines: [C\\_get\\_Covariance 35d, 36a, 39a, 44, 45, 51, 54, 71, 149a](#).

Uses: [Covariance\\_SLOT 21b, C\\_get\\_P 141c, C\\_get\\_Q 142a, C\\_get\\_Variance 143b, C\\_get\\_varonly 142b, LECV 141b](#).

$\langle C\_get\_ExpectationX\ 144a \rangle \equiv$

```
double* C_get_ExpectationX
(
   $\langle R\ LECV\ Input\ 141b \rangle$ 
) {
  return(REAL(VECTOR_ELT(LECV, ExpectationX_SLOT)));
}
◇
```

Fragment referenced in [141a](#).

Defines: `C_get_ExpectationX` [34a](#), [46](#), [71](#).

Uses: `ExpectationX_SLOT` [21b](#), `LECV` [141b](#).

$\langle C\_get\_ExpectationInfluence\ 144b \rangle \equiv$

```
double* C_get_ExpectationInfluence
(
   $\langle R\ LECV\ Input\ 141b \rangle$ 
) {
  return(REAL(VECTOR_ELT(LECV, ExpectationInfluence_SLOT)));
}
◇
```

Fragment referenced in [141a](#).

Defines: `C_get_ExpectationInfluence` [34a](#), [46](#), [149a](#).

Uses: `ExpectationInfluence_SLOT` [21b](#), `LECV` [141b](#).

$\langle C\_get\_CovarianceInfluence\ 144c \rangle \equiv$

```
double* C_get_CovarianceInfluence
(
   $\langle R\ LECV\ Input\ 141b \rangle$ 
) {
  return(REAL(VECTOR_ELT(LECV, CovarianceInfluence_SLOT)));
}
◇
```

Fragment referenced in [141a](#).

Defines: `C_get_CovarianceInfluence` [34a](#), [44](#), [71](#), [149a](#).

Uses: `CovarianceInfluence_SLOT` [21b](#), `LECV` [141b](#).

$\langle C\_get\_VarianceInfluence\ 144d \rangle \equiv$

```
double* C_get_VarianceInfluence
(
   $\langle R\ LECV\ Input\ 141b \rangle$ 
) {
  return(REAL(VECTOR_ELT(LECV, VarianceInfluence_SLOT)));
}
◇
```

Fragment referenced in [141a](#).

Defines: `C_get_VarianceInfluence` [34a](#), [44](#), [71](#), [149a](#).

Uses: `LECV` [141b](#), `VarianceInfluence_SLOT` [21b](#).

$\langle C\_get\_TableBlock\ 145a \rangle \equiv$

```
double* C_get_TableBlock
(
   $\langle R\ LECV\ Input\ 141b \rangle$ 
) {
  if (VECTOR_ELT(LECV, TableBlock_SLOT) == R_NilValue)
    error("object does not contain table block slot");
  return(REAL(VECTOR_ELT(LECV, TableBlock_SLOT)));
}
◇
```

Fragment referenced in [141a](#).

Defines: [C\\_get\\_TableBlock 34a](#).

Uses: [block 26f](#), [27b](#), [LECV 141b](#), [TableBlock\\_SLOT 21b](#).

$\langle C\_get\_Sumweights\ 145b \rangle \equiv$

```
double* C_get_Sumweights
(
   $\langle R\ LECV\ Input\ 141b \rangle$ 
) {
  if (VECTOR_ELT(LECV, Sumweights_SLOT) == R_NilValue)
    error("object does not contain sumweights slot");
  return(REAL(VECTOR_ELT(LECV, Sumweights_SLOT)));
}
◇
```

Fragment referenced in [141a](#).

Defines: [C\\_get\\_Sumweights 34a](#), [46](#).

Uses: [LECV 141b](#), [sumweights 25e](#), [Sumweights\\_SLOT 21b](#).

$\langle C\_get\_Table\ 145c \rangle \equiv$

```
double* C_get_Table
(
   $\langle R\ LECV\ Input\ 141b \rangle$ 
) {
  if (LENGTH(LECV) <= Table_SLOT)
    error("Cannot extract table from object");
  return(REAL(VECTOR_ELT(LECV, Table_SLOT)));
}
◇
```

Fragment referenced in [141a](#).

Defines: [C\\_get\\_Table 41a](#), [46](#).

Uses: [LECV 141b](#), [Table\\_SLOT 21b](#).

$\langle C\_get\_dimTable\ 145d \rangle \equiv$

```
int* C_get_dimTable
(
   $\langle R\ LECV\ Input\ 141b \rangle$ 
) {
  if (LENGTH(LECV) <= Table_SLOT)
    error("Cannot extract table from object");
  return(INTEGER(getAttrib(VECTOR_ELT(LECV, Table_SLOT),
    R_DimSymbol)));
}
◇
```

Fragment referenced in [141a](#).

Defines: [C\\_get\\_dimTable 46](#), [146a](#).

Uses: [LECV 141b](#), [Table\\_SLOT 21b](#).

$\langle C\_get\_B\ 146a \rangle \equiv$

```
int C_get_B
(
   $\langle R\ LECV\ Input\ 141b \rangle$ 
) {
  if (VECTOR_ELT(LECV, TableBlock_SLOT) != R_NilValue)
    return(LENGTH(VECTOR_ELT(LECV, Sumweights_SLOT)));
  return(C_get_dimTable(LECV)[2]);
}
◇
```

Fragment referenced in 141a.

Defines: C\_get\_B 33a, 46, 71.

Uses: C\_get\_dimTable 145d, LECV 141b, Sumweights\_SLOT 21b, TableBlock\_SLOT 21b.

$\langle C\_get\_nresample\ 146b \rangle \equiv$

```
R_xlen_t C_get_nresample
(
   $\langle R\ LECV\ Input\ 141b \rangle$ 
) {
  int PQ = C_get_P(LECV) * C_get_Q(LECV);
  return(XLENGTH(VECTOR_ELT(LECV, PermutedLinearStatistic_SLOT)) / PQ);
}
◇
```

Fragment referenced in 141a.

Defines: C\_get\_nresample 39a, 51, 52a, 54, 56, 71.

Uses: C\_get\_P 141c, C\_get\_Q 142a, LECV 141b, PermutedLinearStatistic\_SLOT 21b.

$\langle C\_get\_PermutedLinearStatistic\ 146c \rangle \equiv$

```
double* C_get_PermutedLinearStatistic
(
   $\langle R\ LECV\ Input\ 141b \rangle$ 
) {
  return(REAL(VECTOR_ELT(LECV, PermutedLinearStatistic_SLOT)));
}
◇
```

Fragment referenced in 141a.

Defines: C\_get\_PermutedLinearStatistic 39a, 51, 71.

Uses: LECV 141b, PermutedLinearStatistic\_SLOT 21b.

$\langle C\_get\_tol\ 146d \rangle \equiv$

```
double C_get_tol
(
   $\langle R\ LECV\ Input\ 141b \rangle$ 
) {
  return(REAL(VECTOR_ELT(LECV, tol_SLOT))[0]);
}
◇
```

Fragment referenced in 141a.

Defines: C\_get\_tol 39a, 51, 54, 71.

Uses: LECV 141b, tol\_SLOT 21b.

⟨ *Memory Input Checks 147a* ⟩ ≡

```
if (P <= 0)
    error("P is not positive");

if (Q <= 0)
    error("Q is not positive");

if (B <= 0)
    error("B is not positive");

if (varonly < 0 || varonly > 1)
    error("varonly is not 0 or 1");

if (Xfactor < 0 || Xfactor > 1)
    error("Xfactor is not 0 or 1");

if (tol <= DBL_MIN)
    error("tol is not positive");
◇
```

Fragment referenced in [148](#).  
Uses: [B 27a](#), [P 24a](#), [Q 24e](#).

⟨ *Memory Names 147b* ⟩ ≡

```
PROTECT(names = allocVector(STRSXP, Table_SLOT + 1));
SET_STRING_ELT(names, LinearStatistic_SLOT, mkChar("LinearStatistic"));
SET_STRING_ELT(names, Expectation_SLOT, mkChar("Expectation"));
SET_STRING_ELT(names, varonly_SLOT, mkChar("varonly"));
SET_STRING_ELT(names, Variance_SLOT, mkChar("Variance"));
SET_STRING_ELT(names, Covariance_SLOT, mkChar("Covariance"));
SET_STRING_ELT(names, ExpectationX_SLOT, mkChar("ExpectationX"));
SET_STRING_ELT(names, dim_SLOT, mkChar("dimension"));
SET_STRING_ELT(names, ExpectationInfluence_SLOT,
    mkChar("ExpectationInfluence"));
SET_STRING_ELT(names, Xfactor_SLOT, mkChar("Xfactor"));
SET_STRING_ELT(names, CovarianceInfluence_SLOT,
    mkChar("CovarianceInfluence"));
SET_STRING_ELT(names, VarianceInfluence_SLOT,
    mkChar("VarianceInfluence"));
SET_STRING_ELT(names, TableBlock_SLOT, mkChar("TableBlock"));
SET_STRING_ELT(names, Sumweights_SLOT, mkChar("Sumweights"));
SET_STRING_ELT(names, PermutedLinearStatistic_SLOT,
    mkChar("PermutedLinearStatistic"));
SET_STRING_ELT(names, StandardisedPermutedLinearStatistic_SLOT,
    mkChar("StandardisedPermutedLinearStatistic"));
SET_STRING_ELT(names, tol_SLOT, mkChar("tol"));
SET_STRING_ELT(names, Table_SLOT, mkChar("Table"));
◇
```

Fragment referenced in [148](#).  
Uses: [CovarianceInfluence\\_SLOT 21b](#), [Covariance\\_SLOT 21b](#), [dim\\_SLOT 21b](#), [ExpectationInfluence\\_SLOT 21b](#),  
[ExpectationX\\_SLOT 21b](#), [Expectation\\_SLOT 21b](#), [LinearStatistic\\_SLOT 21b](#), [PermutedLinearStatistic\\_SLOT 21b](#),  
[StandardisedPermutedLinearStatistic\\_SLOT 21b](#), [Sumweights\\_SLOT 21b](#), [TableBlock\\_SLOT 21b](#), [Table\\_SLOT 21b](#),  
[tol\\_SLOT 21b](#), [VarianceInfluence\\_SLOT 21b](#), [Variance\\_SLOT 21b](#), [varonly\\_SLOT 21b](#), [Xfactor\\_SLOT 21b](#).

$\langle R\_init\_LECV\ 148 \rangle \equiv$

```
SEXP vo, d, names, tolerance, tmp;
int PQ;

 $\langle$  Memory Input Checks 147a  $\rangle$ 
PQ = mPQB(P, Q, 1);
 $\langle$  Memory Names 147b  $\rangle$ 

/* Table_SLOT is always last and only used in 2d case, ie omitted here */
PROTECT(ans = allocVector(VECSXP, Table_SLOT + 1));
SET_VECTOR_ELT(ans, LinearStatistic_SLOT, allocVector(REALSXP, PQ));
SET_VECTOR_ELT(ans, Expectation_SLOT, allocVector(REALSXP, PQ));
SET_VECTOR_ELT(ans, varonly_SLOT, vo = allocVector(INTSXP, 1));
INTEGER(vo)[0] = varonly;
if (varonly) {
    SET_VECTOR_ELT(ans, Variance_SLOT, tmp = allocVector(REALSXP, PQ));
} else {
    /* always return variance */
    SET_VECTOR_ELT(ans, Variance_SLOT, tmp = allocVector(REALSXP, PQ));
    SET_VECTOR_ELT(ans, Covariance_SLOT,
        tmp = allocVector(REALSXP, PP12(PQ)));
}
SET_VECTOR_ELT(ans, ExpectationX_SLOT, allocVector(REALSXP, P));
SET_VECTOR_ELT(ans, dim_SLOT, d = allocVector(INTSXP, 2));
INTEGER(d)[0] = P;
INTEGER(d)[1] = Q;
SET_VECTOR_ELT(ans, ExpectationInfluence_SLOT,
    tmp = allocVector(REALSXP, B * Q));
for (int q = 0; q < B * Q; q++) REAL(tmp)[q] = 0.0;

/* should always _both_ be there */
SET_VECTOR_ELT(ans, VarianceInfluence_SLOT,
    tmp = allocVector(REALSXP, B * Q));
for (int q = 0; q < B * Q; q++) REAL(tmp)[q] = 0.0;

SET_VECTOR_ELT(ans, CovarianceInfluence_SLOT,
    tmp = allocVector(REALSXP, B * Q * (Q + 1) / 2));
for (int q = 0; q < B * Q * (Q + 1) / 2; q++) REAL(tmp)[q] = 0.0;

SET_VECTOR_ELT(ans, Xfactor_SLOT, allocVector(INTSXP, 1));
INTEGER(VECTOR_ELT(ans, Xfactor_SLOT))[0] = Xfactor;
SET_VECTOR_ELT(ans, TableBlock_SLOT, tmp = allocVector(REALSXP, B + 1));
for (int q = 0; q < B + 1; q++) REAL(tmp)[q] = 0.0;
SET_VECTOR_ELT(ans, Sumweights_SLOT, allocVector(REALSXP, B));
SET_VECTOR_ELT(ans, PermutedLinearStatistic_SLOT,
    allocMatrix(REALSXP, 0, 0));
SET_VECTOR_ELT(ans, StandardisedPermutedLinearStatistic_SLOT,
    allocMatrix(REALSXP, 0, 0));
SET_VECTOR_ELT(ans, tol_SLOT, tolerance = allocVector(REALSXP, 1));
REAL(tolerance)[0] = tol;
namesgets(ans, names);

 $\langle$  Initialise Zero 149a  $\rangle$ 
 $\diamond$ 
```

Fragment referenced in 149b, 150.

Uses: B 27a, CovarianceInfluence\_SLOT 21b, Covariance\_SLOT 21b, dim\_SLOT 21b, ExpectationInfluence\_SLOT 21b, ExpectationX\_SLOT 21b, Expectation\_SLOT 21b, LinearStatistic\_SLOT 21b, mPQB 132a, P 24a, PermutedLinearStatistic\_SLOT 21b, PP12 131b, Q 24e, StandardisedPermutedLinearStatistic\_SLOT 21b, Sumweights\_SLOT 21b, TableBlock\_SLOT 21b, Table\_SLOT 21b, tol\_SLOT 21b, VarianceInfluence\_SLOT 21b, Variance\_SLOT 21b, varonly\_SLOT 21b, Xfactor\_SLOT 21b.

*< Initialise Zero 149a >* ≡

```
/* set initial zeros */
for (int p = 0; p < PQ; p++) {
    C_get_LinearStatistic(ans)[p] = 0.0;
    C_get_Expectation(ans)[p] = 0.0;
    if (varonly)
        C_get_Variance(ans)[p] = 0.0;
}
if (!varonly) {
    for (int p = 0; p < PP12(PQ); p++)
        C_get_Covariance(ans)[p] = 0.0;
}
for (int q = 0; q < Q; q++) {
    C_get_ExpectationInfluence(ans)[q] = 0.0;
    C_get_VarianceInfluence(ans)[q] = 0.0;
}
for (int q = 0; q < Q * (Q + 1) / 2; q++)
    C_get_CovarianceInfluence(ans)[q] = 0.0;
◇
```

Fragment referenced in [148](#).

Uses: [C\\_get\\_Covariance 143c](#), [C\\_get\\_CovarianceInfluence 144c](#), [C\\_get\\_Expectation 143a](#),  
[C\\_get\\_ExpectationInfluence 144b](#), [C\\_get\\_LinearStatistic 142d](#), [C\\_get\\_Variance 143b](#),  
[C\\_get\\_VarianceInfluence 144d](#), [PP12 131b](#), [Q 24e](#).

*< RC\_init\_LECV\_1d 149b >* ≡

```
SEXP RC_init_LECV_1d
(
    < C integer P Input 24a >,
    < C integer Q Input 24e >,
    int varonly,
    < C integer B Input 27a >,
    int Xfactor,
    double tol
) {
    SEXP ans;

    < R_init_LECV 148 >

    SET_VECTOR_ELT(ans, TableBlock_SLOT,
        allocVector(REALSXP, B + 1));

    SET_VECTOR_ELT(ans, Sumweights_SLOT,
        allocVector(REALSXP, B));

    UNPROTECT(2);
    return(ans);
}
◇
```

Fragment referenced in [141a](#).

Defines: [RC\\_init\\_LECV\\_1d 31a](#).

Uses: [B 27a](#), [Sumweights\\_SLOT 21b](#), [TableBlock\\_SLOT 21b](#).

$\langle RC\_init\_LECV\_2d\ 150 \rangle \equiv$

```
SEXP RC_init_LECV_2d
(
   $\langle C\ integer\ P\ Input\ 24a \rangle$ ,
   $\langle C\ integer\ Q\ Input\ 24e \rangle$ ,
  int varonly,
  int Lx,
  int Ly,
   $\langle C\ integer\ B\ Input\ 27a \rangle$ ,
  int Xfactor,
  double tol
) {
  SEXP ans, tabdim, tab;

  if (Lx <= 0)
    error("Lx is not positive");

  if (Ly <= 0)
    error("Ly is not positive");

   $\langle R\_init\_LECV\ 148 \rangle$ 

  PROTECT(tabdim = allocVector(INTSXP, 3));
  INTEGER(tabdim)[0] = Lx + 1;
  INTEGER(tabdim)[1] = Ly + 1;
  INTEGER(tabdim)[2] = B;
  SET_VECTOR_ELT(ans, Table_SLOT,
                 tab = allocVector(REALSXP,
                                   INTEGER(tabdim)[0] *
                                   INTEGER(tabdim)[1] *
                                   INTEGER(tabdim)[2]));
  dimgets(tab, tabdim);

  UNPROTECT(3);
  return(ans);
}
◇
```

Fragment referenced in [141a](#).  
Defines: [RC\\_init\\_LECV\\_2d 41a](#).  
Uses: [B 27a](#), [Table\\_SLOT 21b](#).

## Chapter 4

# Package Infrastructure

"AAA.R" 151a≡

```
< R Header 154a >
.onUnload <-
function(libpath)
  library.dynam.unload("libcoin", libpath)
◇
```

"DESCRIPTION" 151b≡

```
Package: libcoin
Title: Linear Test Statistics for Permutation Inference
Date: 2023-09-26
Version: 1.0-10
Authors@R: person("Torsten", "Hothorn", role = c("aut", "cre"),
                  email = "Torsten.Hothorn@R-project.org")
Description: Basic infrastructure for linear test statistics and permutation
             inference in the framework of Strasser and Weber (1999) <https://epub.wu.ac.at/102/>.
             This package must not be used by end-users. CRAN package 'coin' implements all
             user interfaces and is ready to be used by anyone.
Depends: R (>= 3.4.0)
Suggests: coin
Imports: stats, mvtnorm
LinkingTo: mvtnorm
NeedsCompilation: yes
License: GPL-2
◇
```

"NAMESPACE" 151c≡

```
useDynLib(libcoin, .registration = TRUE)

importFrom("stats", complete.cases, vcov)
importFrom("mvtnorm", GenzBretz)

export(LinStatExpCov, doTest, ctabs, lmult)

S3method(vcov, LinStatExpCov)
◇
```

Add flag `-g` to `PKG_CFLAGS` for `operf` profiling (this is not portable).

```
"Makevars" 152a≡
```

```
PKG_CFLAGS=$(C_VISIBILITY)
PKG_LIBS = $(LAPACK_LIBS) $(BLAS_LIBS) $(FLIBS)
◇
```

```
"libcoin-win.def" 152b≡
```

```
LIBRARY libcoin.dll
EXPORTS
  R_init_libcoin
◇
```

Other packages can link against **libcoin**. A small example package is contained in `libcoin/inst/C-API_example`.

```
"libcoin-init.c" 152c≡
```

```
{ C Header 154b }
#include "libcoin.h"
#include <R_ext/Rdynload.h>
#include <R_ext/Visibility.h>

#define CALLDEF(name, n) {#name, (DL_FUNC) &name, n}
#define REGCALL(name) R_RegisterCCallable("libcoin", #name, (DL_FUNC) &name)

static const R_CallMethodDef callMethods[] = {
  CALLDEF(R_ExpectationCovarianceStatistic, 7),
  CALLDEF(R_PermutatedLinearStatistic, 6),
  CALLDEF(R_StandardisePermutatedLinearStatistic, 1),
  CALLDEF(R_ExpectationCovarianceStatistic_2d, 9),
  CALLDEF(R_PermutatedLinearStatistic_2d, 7),
  CALLDEF(R_QuadraticTest, 5),
  CALLDEF(R_MaximumTest, 9),
  CALLDEF(R_MaximallySelectedTest, 6),
  CALLDEF(R_ExpectationInfluence, 3),
  CALLDEF(R_CovarianceInfluence, 4),
  CALLDEF(R_ExpectationX, 4),
  CALLDEF(R_CovarianceX, 5),
  CALLDEF(R_Sums, 3),
  CALLDEF(R_KronSums, 6),
  CALLDEF(R_KronSums_Permutation, 5),
  CALLDEF(R_colSums, 3),
  CALLDEF(R_OneTableSums, 3),
  CALLDEF(R_TwoTableSums, 4),
  CALLDEF(R_ThreeTableSums, 5),
  CALLDEF(R_order_subset_wrt_block, 4),
  CALLDEF(R_quadform, 3),
  CALLDEF(R_kronecker, 2),
  CALLDEF(R_MPinv_sym, 3),
  CALLDEF(R_unpack_sym, 3),
  CALLDEF(R_pack_sym, 1),
  {NULL, NULL, 0}
};
◇
```

File defined by [152c](#), [153](#).

Uses: [R\\_colSums 106c](#), [R\\_CovarianceInfluence 83a](#), [R\\_CovarianceX 87a](#), [R\\_ExpectationCovarianceStatistic 31a](#), [R\\_ExpectationCovarianceStatistic\\_2d 41a](#), [R\\_ExpectationInfluence 81a](#), [R\\_ExpectationX 84c](#), [R\\_KronSums 94b](#), [R\\_KronSums\\_Permutation 102b](#), [R\\_MPinv\\_sym 136b](#), [R\\_OneTableSums 111a](#), [R\\_order\\_subset\\_wrt\\_block 123c](#), [R\\_pack\\_sym 140c](#), [R\\_PermutatedLinearStatistic 37](#), [R\\_PermutatedLinearStatistic\\_2d 48](#), [R\\_quadform 61b](#), [R\\_Sums 90a](#), [R\\_ThreeTableSums 119b](#), [R\\_TwoTableSums 115a](#), [R\\_unpack\\_sym 139](#).

"libcoin-init.c" 153≡

```
void attribute_visible R_init_libcoin
(
    DllInfo *dll
) {
    R_registerRoutines(dll, NULL, callMethods, NULL, NULL);
    R_useDynamicSymbols(dll, FALSE);
    R_forceSymbols(dll, TRUE);
    REGCALL(R_ExpectationCovarianceStatistic);
    REGCALL(R_PermutatedLinearStatistic);
    REGCALL(R_StandardisePermutatedLinearStatistic);
    REGCALL(R_ExpectationCovarianceStatistic_2d);
    REGCALL(R_PermutatedLinearStatistic_2d);
    REGCALL(R_QuadraticTest);
    REGCALL(R_MaximumTest);
    REGCALL(R_MaximallySelectedTest);
    REGCALL(R_ExpectationInfluence);
    REGCALL(R_CovarianceInfluence);
    REGCALL(R_ExpectationX);
    REGCALL(R_CovarianceX);
    REGCALL(R_Sums);
    REGCALL(R_KronSums);
    REGCALL(R_KronSums_Permutation);
    REGCALL(R_colSums);
    REGCALL(R_OneTableSums);
    REGCALL(R_TwoTableSums);
    REGCALL(R_ThreeTableSums);
    REGCALL(R_order_subset_wrt_block);
    REGCALL(R_quadform);
    REGCALL(R_kronecker);
    REGCALL(R_MPinv_sym);
    REGCALL(R_unpack_sym);
    REGCALL(R_pack_sym);
}
◇
```

File defined by 152c, 153.

Uses: R\_colSums 106c, R\_CovarianceInfluence 83a, R\_CovarianceX 87a, R\_ExpectationCovarianceStatistic 31a, R\_ExpectationCovarianceStatistic\_2d 41a, R\_ExpectationInfluence 81a, R\_ExpectationX 84c, R\_KronSums 94b, R\_KronSums\_Permutation 102b, R\_MPinv\_sym 136b, R\_OneTableSums 111a, R\_order\_subset\_wrt\_block 123c, R\_pack\_sym 140c, R\_PermutatedLinearStatistic 37, R\_PermutatedLinearStatistic\_2d 48, R\_quadform 61b, R\_Sums 90a, R\_ThreeTableSums 119b, R\_TwoTableSums 115a, R\_unpack\_sym 139.

*< R Header 154a >* ≡

```
### Copyright (C) 2017-2023 Torsten Hothorn
###
### This file is part of the 'libcoin' R add-on package.
###
### 'libcoin' is free software: you can redistribute it and/or modify
### it under the terms of the GNU General Public License as published by
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###
### 'libcoin' is distributed in the hope that it will be useful,
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###
### You should have received a copy of the GNU General Public License
### along with 'libcoin'. If not, see <http://www.gnu.org/licenses/>.
###
### DO NOT EDIT THIS FILE
###
### Edit 'libcoin.w' and run 'nuweb -r libcoin.w'
◇
```

Fragment referenced in [3a](#), [15b](#), [151a](#).

*< C Header 154b >* ≡

```
/*
Copyright (C) 2017-2023 Torsten Hothorn

This file is part of the 'libcoin' R add-on package.

'libcoin' is free software: you can redistribute it and/or modify
it under the terms of the GNU General Public License as published by
the Free Software Foundation, version 2.

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along with 'libcoin'. If not, see <http://www.gnu.org/licenses/>.

DO NOT EDIT THIS FILE

Edit 'libcoin.w' and run 'nuweb -r libcoin.w'
*/
◇
```

Fragment referenced in [20a](#), [22ac](#), [30d](#), [152c](#).

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